

# Refinement Checking for Libraries of Concurrent Objects

Ahmed Bouajjani

LIAFA, Univ Paris Diderot - Paris 7

Joint work with

Michael Emmi

IMDEA

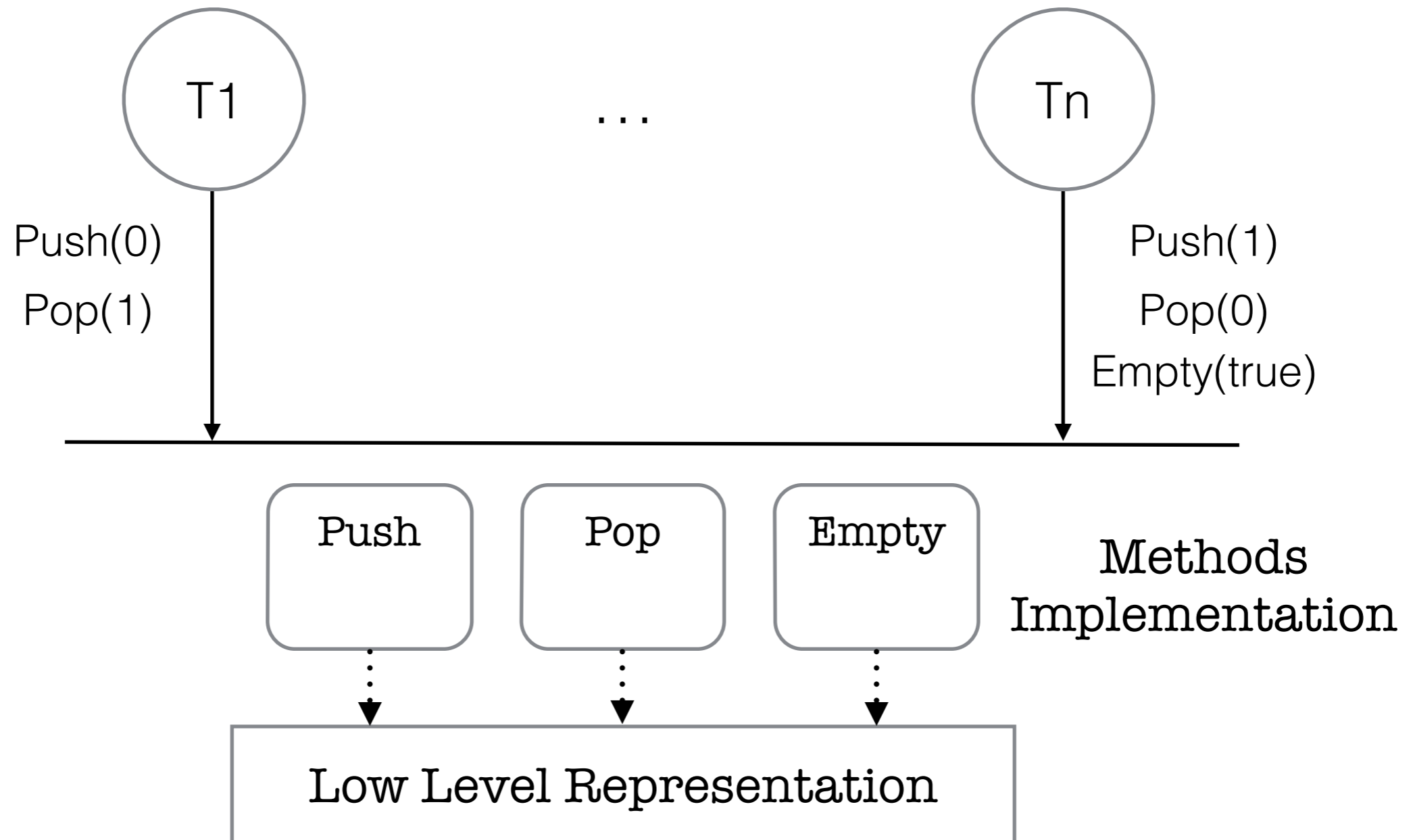
Constantin Enea

LIAFA, U Paris Diderot - P7

Jad Hamza

PV, Madrid, September 5, 2015

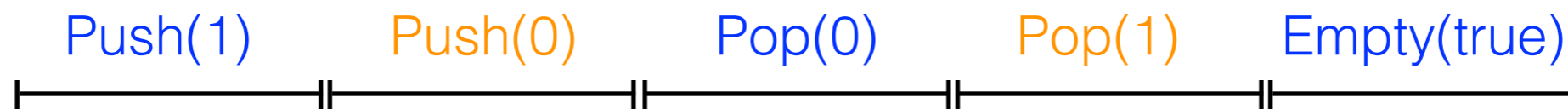
# Concurrent Data Structures



# Different atomicity levels

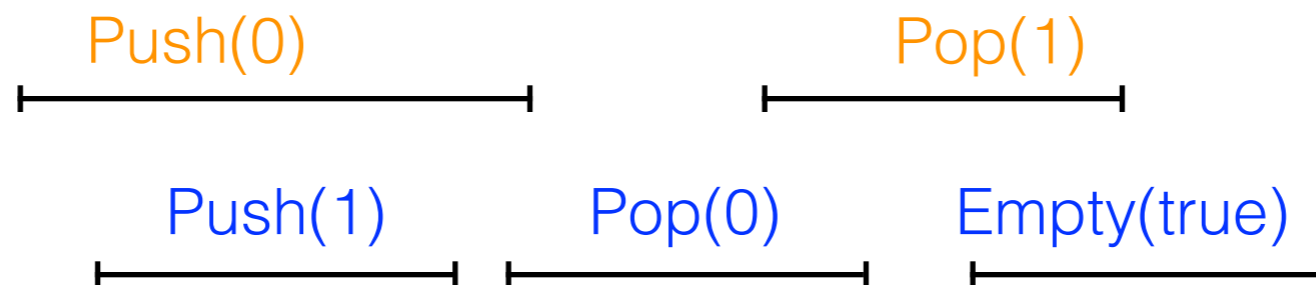
## Client view:

- Operations are **atomic**
- Thread executions are interleaved

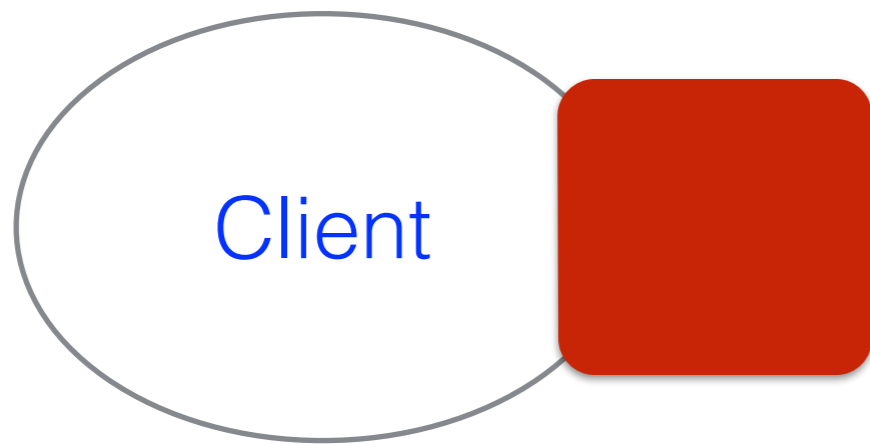


## Implementation:

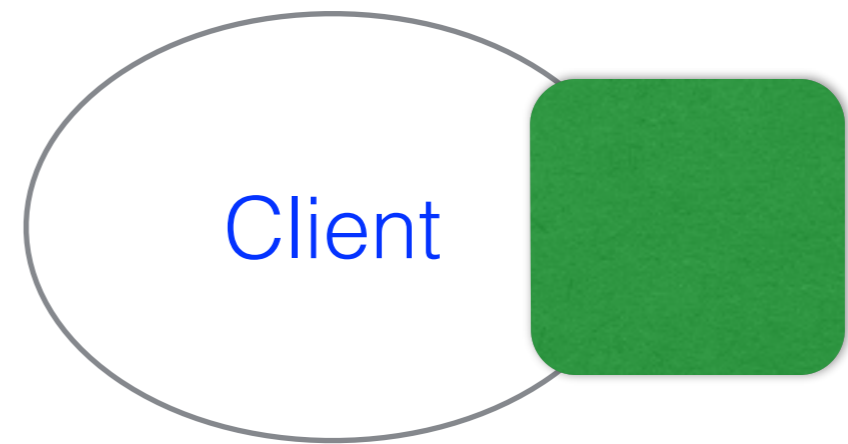
- **Naive solution:** Coarse-grain locking
- **Performances:** Avoid coarse-grain locking
- => **Execution intervals may overlap**



# Observational Refinement



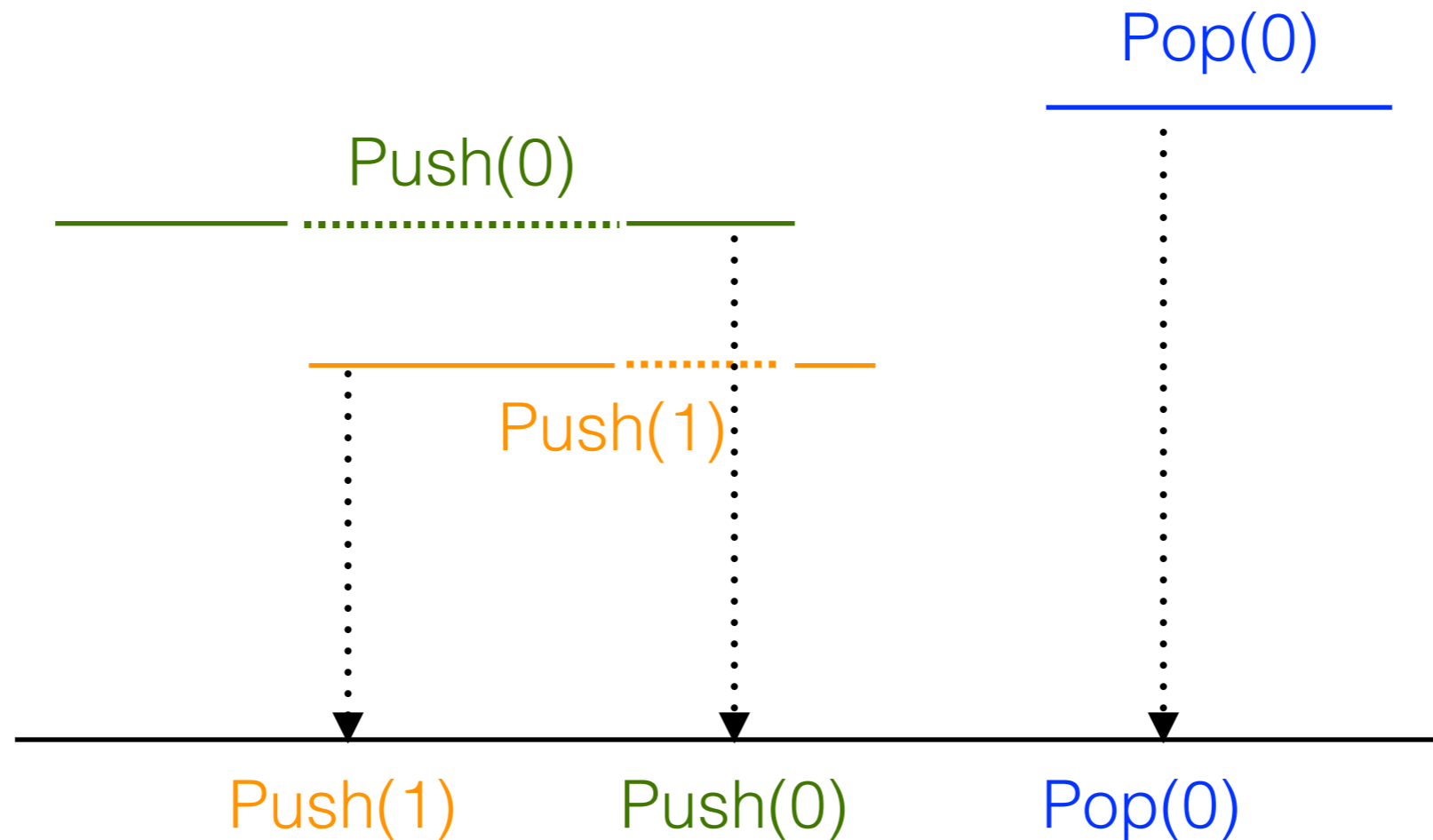
Implementation



Specification:  
Atomic Operations

For every Client,  
Client x Impl included in Client x Spec

# Linearizability [Herlihy, Wing, 1990]



Valid sequence in the sequential specification

- Reorder call/return events, while preserving returns  $\rightarrow$  calls
- Find “linearization points” within execution time intervals
- $\Rightarrow$  match a sequential execution

Linearizability *implies* Observational Refinement

[Filipovic, O’Hearn, Rinetzky, Yang, 2009]

# Complexity

- Checking linearizability of a **single execution** is **NP-complete**  
[Gibbons, Korach, 1997]
- For finite-state implementations and specifications:

	Linearizability	State Reachability
Fixed Nb Threads	EXPSPACE-complete (1)	PSPACE-complete
Unbounded Nb of Threads	Undecidable (2)	EXPSPACE-complete

(1) Upper Bound: Alur, McMillan, Peled 1996 — Lower Bound: Hamza 2015

(2) B., Emmi, Enea, Hamza, 2013

# Reducing Linearizability/OR to State Reachability?

## Why?

- Reuse existing tools for SR
- Lower complexity
- Decidability

## Two approaches:

- **Under approximations:** special classes of executions
  - Parametrized under-approximation schema?
  - Good coverage, low complexity, scalability
- **Special classes of objects:**  
Stacks, queues, etc.

# Under-approximate detection of OR violations ?

[B, Emmi, Enea, Hamza, POPL'15]

## Characterizing OR as a History Inclusion Problem

- “Histories” are a special partial orders: Interval orders
- => Interval-length bounding

## Efficient Reduction to Reachability

- Use counting representations
- => Correctness of a single history is Polynomial time
- => Scalable dynamic and static analysis techniques

## Good coverage with small bounds

- $\leq 3$
- Cut-off bounds for common data structures



# Histories

History of an execution  $e$  :

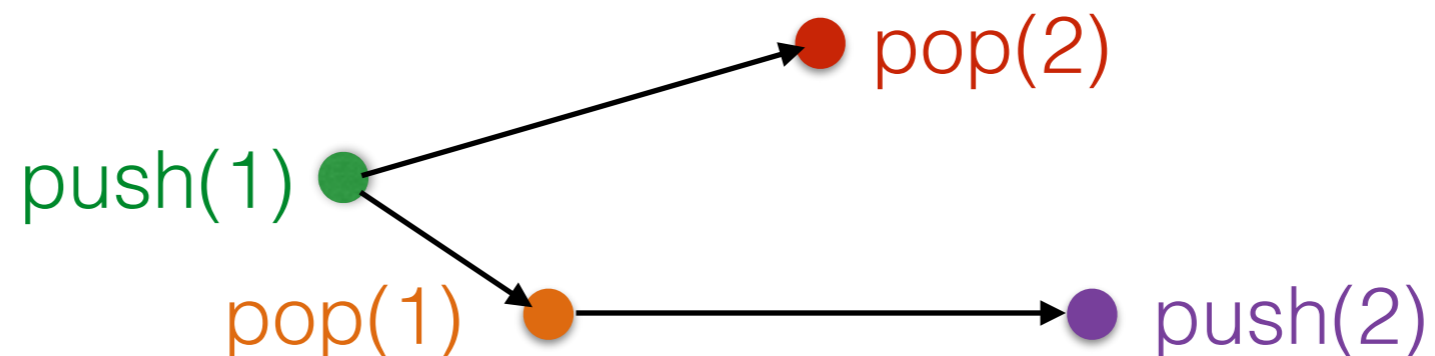
$$H(e) = (O, \text{label}, <)$$

where

- $O = \text{Operations}(e)$
- $\text{label}: O \rightarrow M \times V \times V$
- $<$  is a partial order s.t.

$O_1 < O_2$  iff  $\text{Return}(O_1)$  is *before*  $\text{Call}(O_2)$  in  $e$

$c(\text{push}, 1)$   $r(\text{push}, \text{tt})$   $c(\text{pop}, -)$   $c(\text{pop}, -)$   $r(\text{pop}, 1)$   $c(\text{push}, 2)$   $r(\text{push}, \text{tt})$   $r(\text{pop}, 2)$



# Abstracting Histories

## Weakening relation

$h_1 \leq h_2$  ( $h_1$  is weaker than  $h_2$ )  
iff

$h_1$  has less constraints than  $h_2$

## Lemma:

If  $h_1 \leq h_2$  and  $h_2$  is in  $H(L)$ , then  $h_1$  is in  $H(L)$  too

# Approximation Schema for detecting OR violations

Parametrized weakening function  $A_k$ , for any  $k \geq 0$ , s.t.

- $A_k(h) \leq h$
- $A_0(h) \leq A_1(h) \leq A_2(h) \leq \dots \leq h$
- There is a  $k$  s.t.  $h \leq A_k(h)$
- Checking if  $A_k(h)$  is in  $H(L)$  decidable in **polynomial time**

# Approximation Schema for detecting OR violations

Parametrized weakening function  $A_k$ , for any  $k \geq 0$ , s.t.

- $A_k(h) \leq h$
- $A_0(h) \leq A_1(h) \leq A_2(h) \leq \dots \leq h$
- There is a  $k$  s.t.  $h \leq A_k(h)$
- Checking if  $A_k(h)$  is in  $H(L)$  decidable in polynomial time

## Approximating History Inclusion

- Choose a parameter  $k \geq 0$
- Is there an  $h$  in  $H(L_1)$  s.t.  $A_k(h)$  is not in  $H(L_2)$  ?
- **Lemma**  $\Rightarrow$  If  $A_k(h)$  not in  $H(L_2)$ , then  $h$  is not in  $H(L_2)$

# Histories are Interval Orders

Interval Orders = partial order  $(O, <)$  such that

$(o_1 < o_1'$  and  $o_2 < o_2')$  implies  $(o_1 < o_2'$  or  $o_2 < o_1')$



Prop: For every execution  $e$ ,  $H(e)$  is an interval order

# A Bounding Concept for Histories

Let  $h = (O, <)$  be an Interval Order (history in our case)

Notion of length:

- Past of an operation:  $\text{past}(o) = \{o' : o' < o\}$
- Lemma [Rabinovitch'78]:

The set  $\{\text{past}(o) : o \text{ in } O\}$  is *linearly ordered*

- The *length* of the order = number of pasts - 1

# A Bounding Concept for Histories

Let  $h = (O, <)$  be an Interval Order (history in our case)

Notion of length:

- Past of an operation:  $\text{past}(o) = \{o' : o' < o\}$
- Lemma [Rabinovitch'78]:

The set  $\{\text{past}(o) : o \text{ in } O\}$  is *linearly ordered*

- The *length* of the order = number of pasts - 1

Bounded interval-length approximation

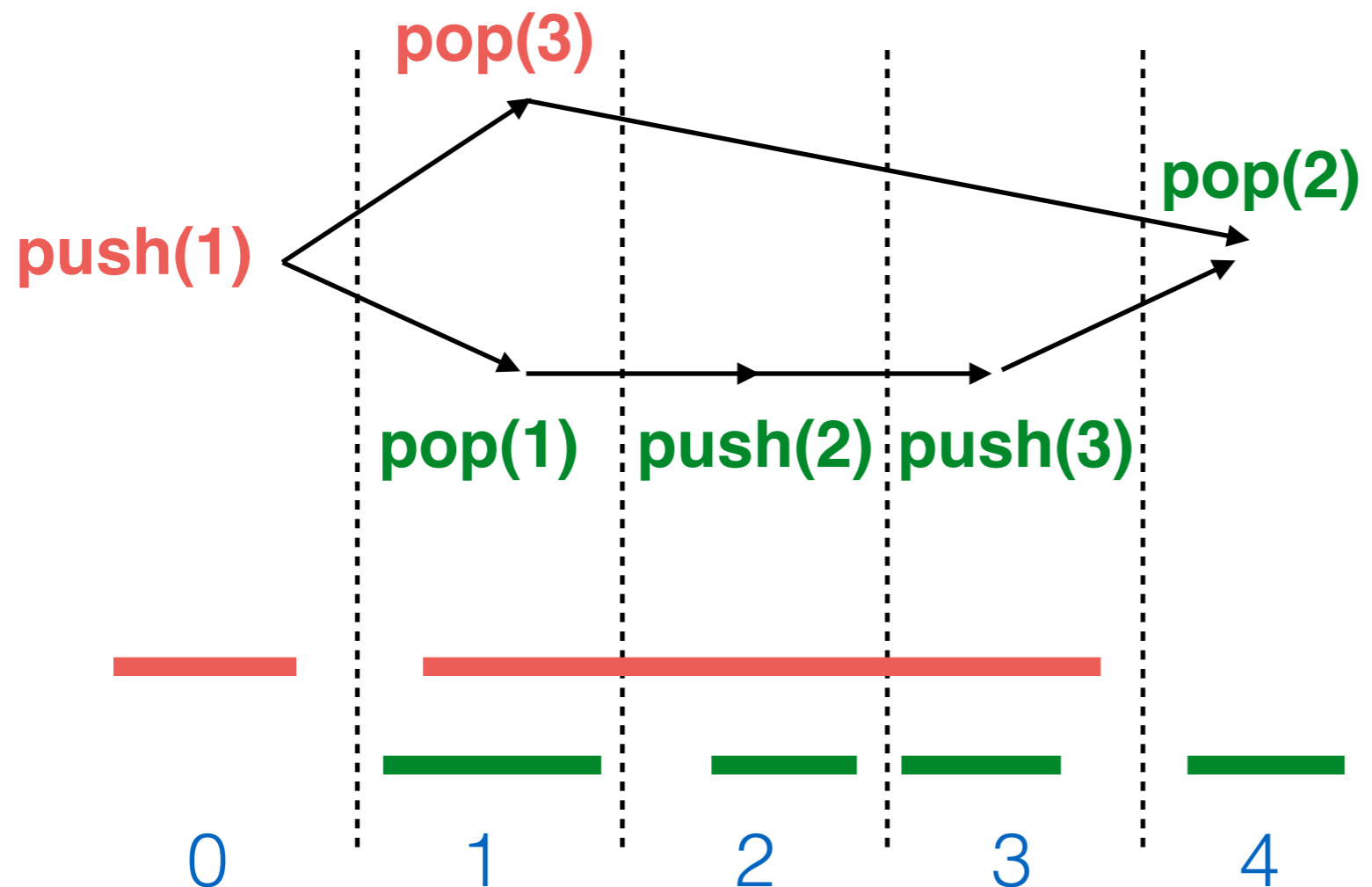
$A_k$  maps each  $h$  to some  $h' \leq h$  of length  $k$

$\Rightarrow A_k$  keeps precise the information (bounds)  
about the  $k$  last intervals

# Canonical Representation of Interval Orders

- Mapping  $I : O \rightarrow [n]^2$  where  $n = \text{length}(h)$  [Greenough '76]
- $I(o) = [i, j]$ , with  $i, j \leq n$ , such that

$$i = |\{\text{past}(o') : o' < o\}| \text{ and} \\ j = |\{\text{past}(o') : \text{not } (o < o')\}| - 1$$



$$\begin{aligned} I(\text{push}(1)) &= [0, 0] \\ I(\text{pop}(1)) &= [1, 1] \\ I(\text{push}(2)) &= [2, 2] \\ I(\text{push}(3)) &= [3, 3] \\ I(\text{pop}(3)) &= [1, 3] \\ I(\text{pop}(2)) &= [4, 4] \end{aligned}$$



# Counting Representation of Interval Orders

Count the number of occurrences of each operation in each interval

- $h = (O, <)$  an IO with canonical representation  $I:O \rightarrow [n]^2$
- Then, let  $\Pi(h)$  be the multi-set  $\{ [\text{label}(o), I(o)] : o \text{ in } O \}$
- Use a counter per type of operation and interval

Prop:  $H(e)$  is in  $H(L)$  iff  $\Pi(H(e))$  is in  $\Pi(H(L))$

# Reduction to Reachability with Counters

$H(L_1)$  subset of  $H(L_2)$

iff

$\Pi(H(L_1))$  subset of  $\Pi(H(L_2))$

- Consider *only*  $k$ -bounded-length histories
- Track histories of  $L_1$  using a finite number of counters
- Use an arithmetic-based representation of  $\Pi(H(L_2))$
- Check that  $\Pi(H(L_2))$  is invariant
- $\Rightarrow$  Dynamic and static analysis

# Reduction to Reachability with Counters

$H(L_1)$  subset of  $H(L_2)$

iff

$\Pi(H(L_1))$  subset of  $\Pi(H(L_2))$

- Consider *only*  $k$ -bounded-length histories
- Track histories of  $L_1$  using a finite number of counters
- Use an arithmetic-based representation of  $\Pi(H(L_2))$
- Check that  $\Pi(H(L_2))$  is invariant
- $\Rightarrow$  Dynamic and static analysis

How to get  $\Pi(H(L))$  ?

# Operation counting formulas

- Logic for expressing operation counting constraints

Presburger arithmetics with  $\#(a, i, j)$  predicates  
where

$$[\#(a, i, j)](h) = |\{o : l(o) = [i, j] \text{ and } \text{label}(o) = a\}|$$

*Number of occurrences of  $a$  in the interval  $[i, j]$*

- $h \models F$  is polynomial time (for a fixed quantifier count)

# Building operation counting formulas

- For any  $k$ , a counting formula for  $\Pi(H(L))$  can be provided for common data structures (**stack, queue, set,...**)
- Assume **data independence**  $\Rightarrow$  push's have  $\neq$  values

Violation of FIFO property (FV):

$$\exists x1, x2, y1, y2.$$

$$\text{Match}(x1, y1) \ \& \ \text{Match}(x2, y2)$$

$$\text{Before}_k(x1, x2) \ \& \ \text{Before}_k(y2, y1)$$

where

$$\text{Match}(x, y) = \text{IsPush}(x) \ \& \ \text{IsPop}(y) \ \& \ \text{SameVal}(x, y)$$

$$\text{Before}_k(x, y) = \bigvee_{i \leq k} (\text{count}(x, 0, i) > 0) \ \& \ \text{count}(y, 0, i) = 0 \ \& \ \text{count}(x, i+1, k) = 0$$

where

$$\text{count}(x, i, j) = \sum_{i \leq i' \leq j' \leq j} \#(x, i', j')$$

# Building operation counting formulas

FIFO queue data structure:

Violation of FIFO property (FV):

$$\exists x_1, x_2, y_1, y_2. \text{Match}(x_1, y_1) \ \& \ \text{Match}(x_2, y_2) \ \& \ \text{Before}_\kappa(x_1, x_2) \ \& \ \text{Before}_\kappa(y_2, y_1)$$

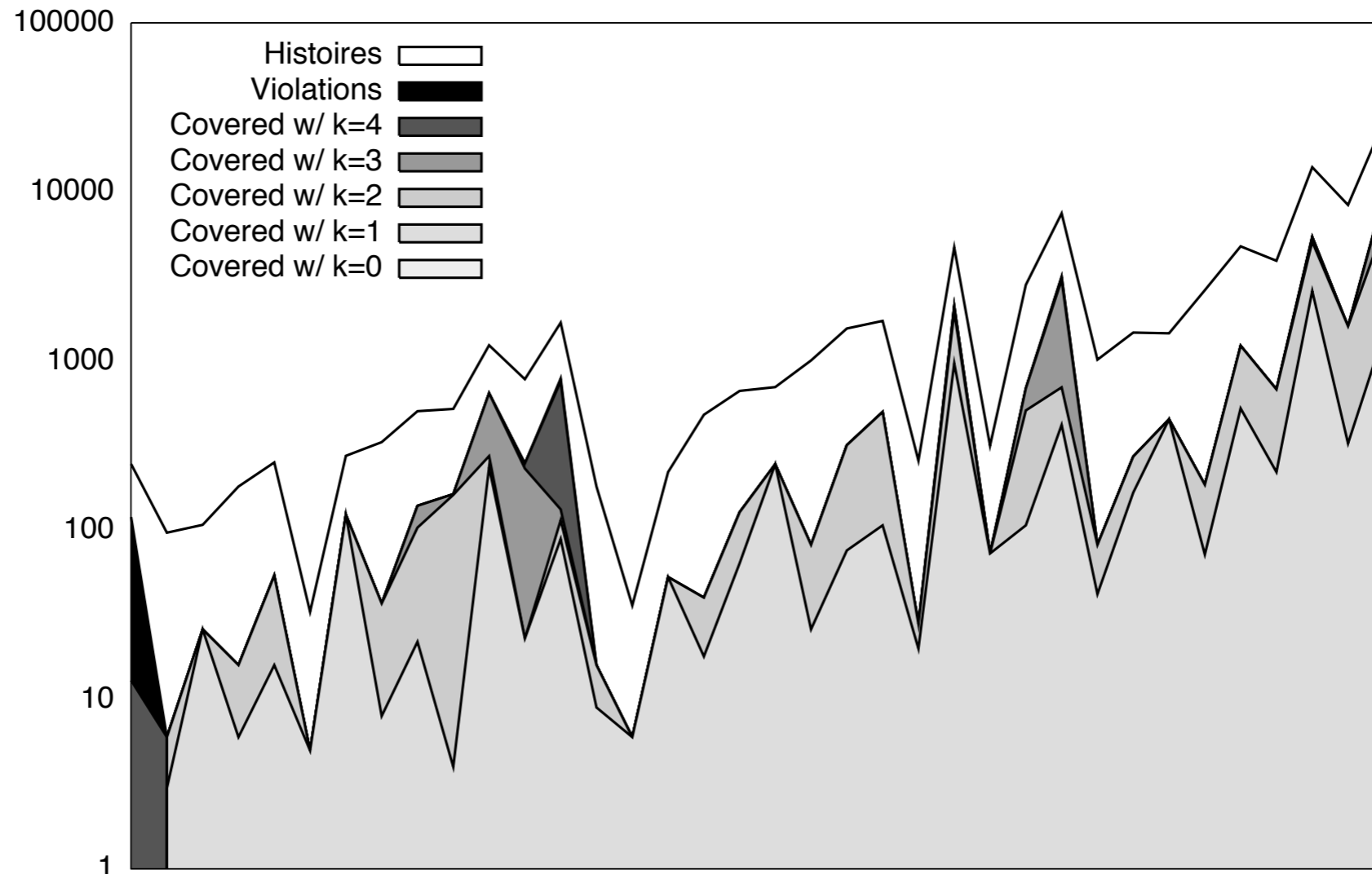
Violation of Remove property (RV):

$$\exists x, y. \text{Match}(x, y) \ \& \ \text{Before}_\kappa(y, x)$$

Violation of Empty property (EV):

$$\exists x, y, z. \text{Match}(x, z) \ \& \ \text{EmptyPop}(y) \ \& \ \text{Before}_\kappa(y, z) \ \& \ \text{Before}_\kappa(y, z)$$

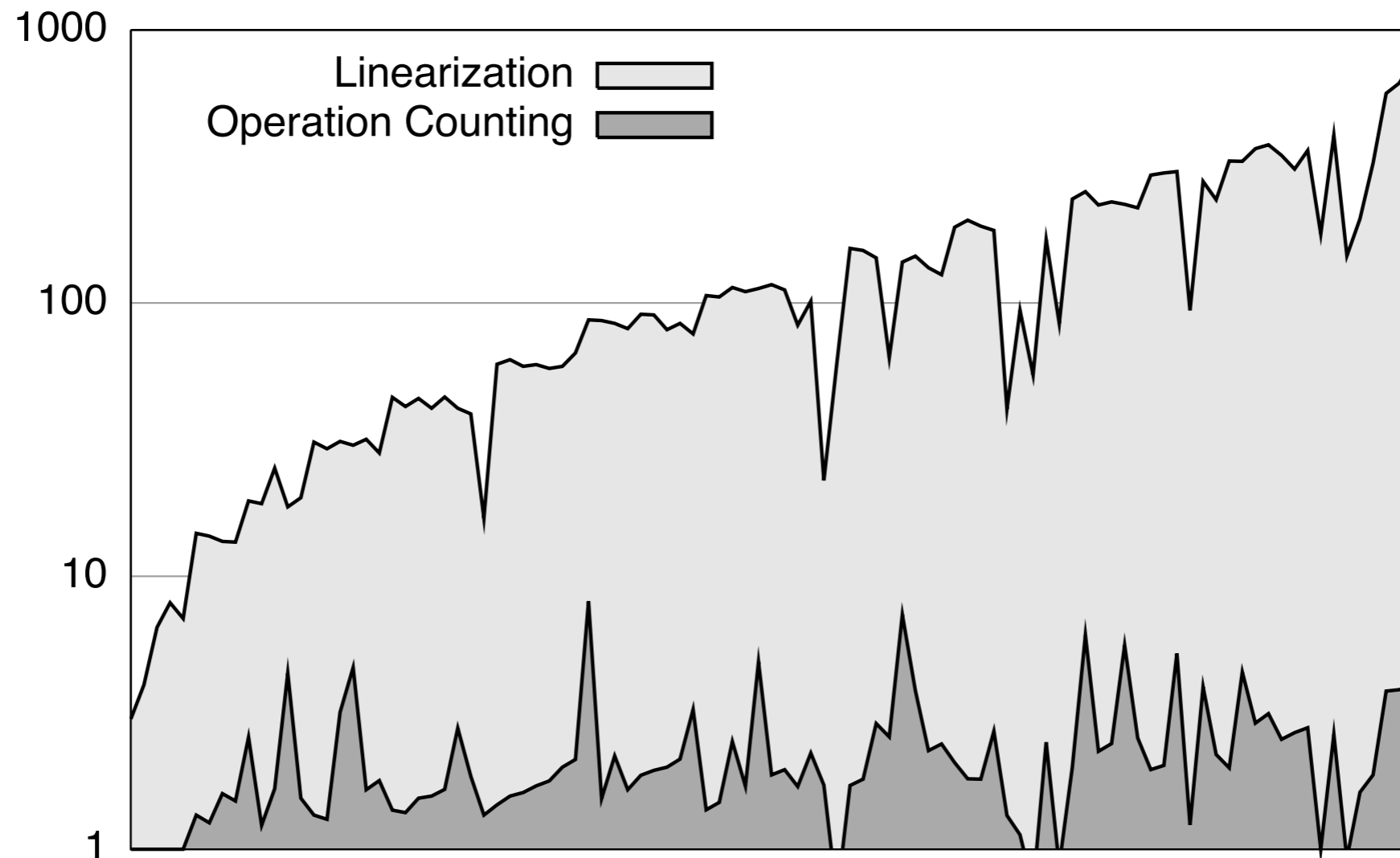
# Experimental Results: Coverage



Comparison of violations covered with  $k \leq 4$

- Data point: Counts in logarithmic scale over all executions (up to 5 preemptions) on Scal's nonblocking bounded-reordering queue with  $\leq 4$  enqueue and  $\leq 4$  dequeue
- x-axis: increasing number of executions (1023-2359292)
- White: total number of unique histories over a given set of executions
- Black: violations detected by traditional linearizability checker (e.g., Line-up)

# Experimental Results: Runtime Monitoring



Comparison of runtime overhead  
between Linearization-based monitoring and Operation counting

- Data point: runtime on logarithmic scale, normalised on unmonitored execution time
- Scal's nonblocking Michael-Scott queue, 10 enqueue and 10 dequeue operations.
- x-axis is ordered by increasing number of operations



# Experimental Results: Static Analysis

<b>Library</b>	<b>Bug</b>	<b>P</b>	<b>k</b>	<b>m</b>	<b>n</b>	<b>Time</b>
Michael-Scott Queue	B1 (head)	2x2	1	2	2	24.76s
Michael-Scott Queue	B1 (tail)	3x1	1	2	3	45.44s
Treiber Stack	B2	3x4	1	1	2	52.59s
Treiber Stack	B3 (push)	2x2	1	1	2	24.46s
Treiber Stack	B3 (pop)	2x2	1	1	2	15.16s
Elimination Stack	B4	4x1	0	1	4	317.79s
Elimination Stack	B5	3x1	1	1	4	222.04s
Elimination Stack	B2	3x4	0	1	2	434.84s
Lock-coupling Set	B6	1x2	0	2	2	11.27s
LFDS Queue	B7	2x2	1	1	2	77.00s

- Static detection of injected refinement violations with CSeq & CBMC.
- Program Pij with i and j invocations to the push and pop methods, explore n-round round-robin schedules with m loop iterations unrolled, with monitor for Ak.
- Bugs: (B1) non-atomic lock, (B2) ABA bug, (B3) non-atomic CAS operation, (B4) misplaced brace, (B5) forgotten assignment, (B6) misplaced

# Focusing on Special Classes of Objects

[B., Emmi, Enea, Hamza, ICALP 2015]

- Inductive definition of sequential objects (restricted language based on constrained rewrite rules)
- Characterizing concurrent violations using a finite number of “bad patterns”
- Defining finite-state automata recognising each of the “bad patterns” (using data independence assumption)
- Reducing linearizability to checking the emptiness of the intersection with these automata.

# Specifying queues and stacks

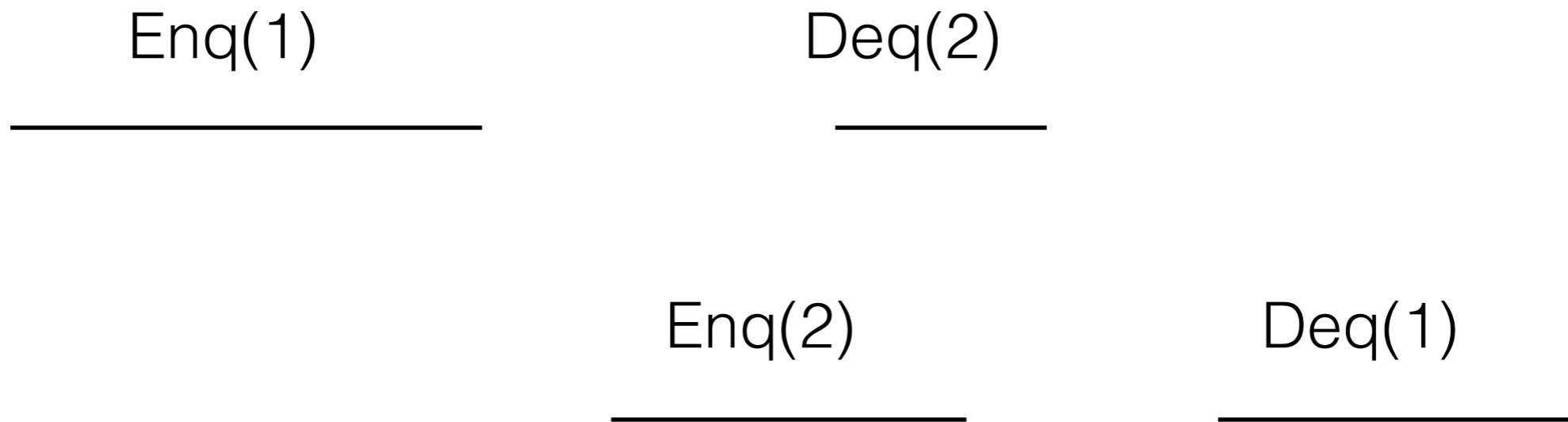
## Queue

- $u . v : Q \ \& \ u : \text{ENQ}^* \longrightarrow \mathbf{Enq(x)} . u . \mathbf{Deq(x)} . v : Q$
- $u . v : Q \ \& \ \text{no unmatched } \textit{Enq} \text{ in } u \longrightarrow u . \mathbf{Emp} . v : Q$

## Stack

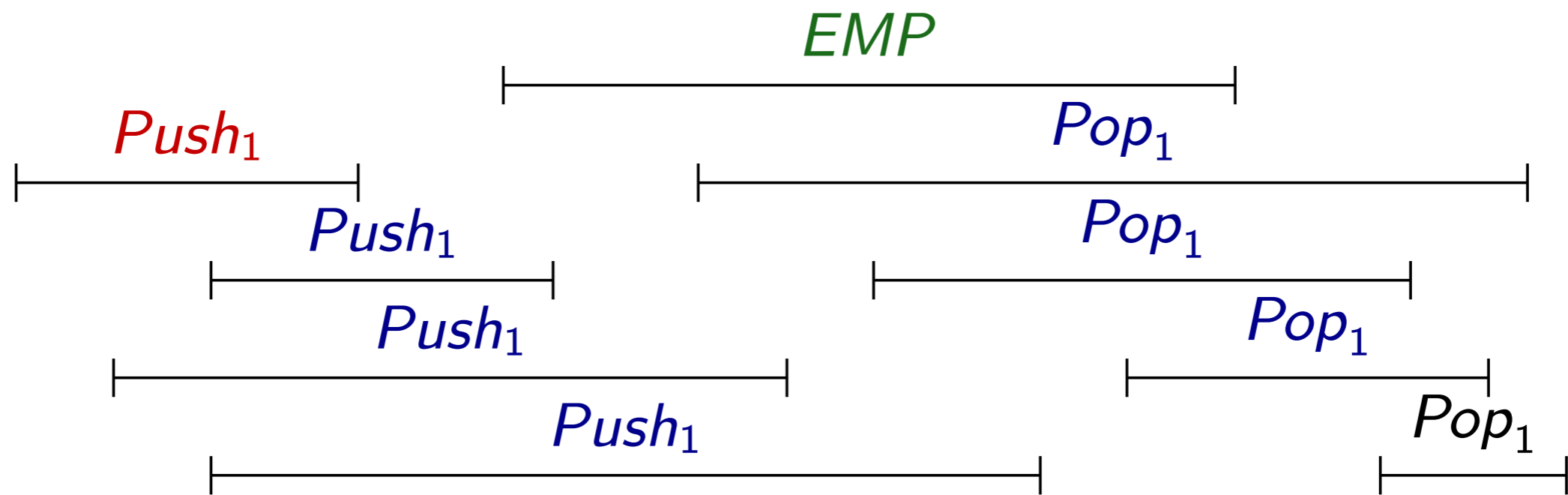
- $u . v : S \ \& \ \text{no unmatched } \textit{Push} \text{ in } u \longrightarrow \mathbf{Push(x)} . u . \mathbf{Pop(x)} . v : S$
- $u . v : S \ \& \ \text{no unmatched } \textit{Push} \text{ in } u \longrightarrow u . \mathbf{Emp} . v : S$

# Order Violation

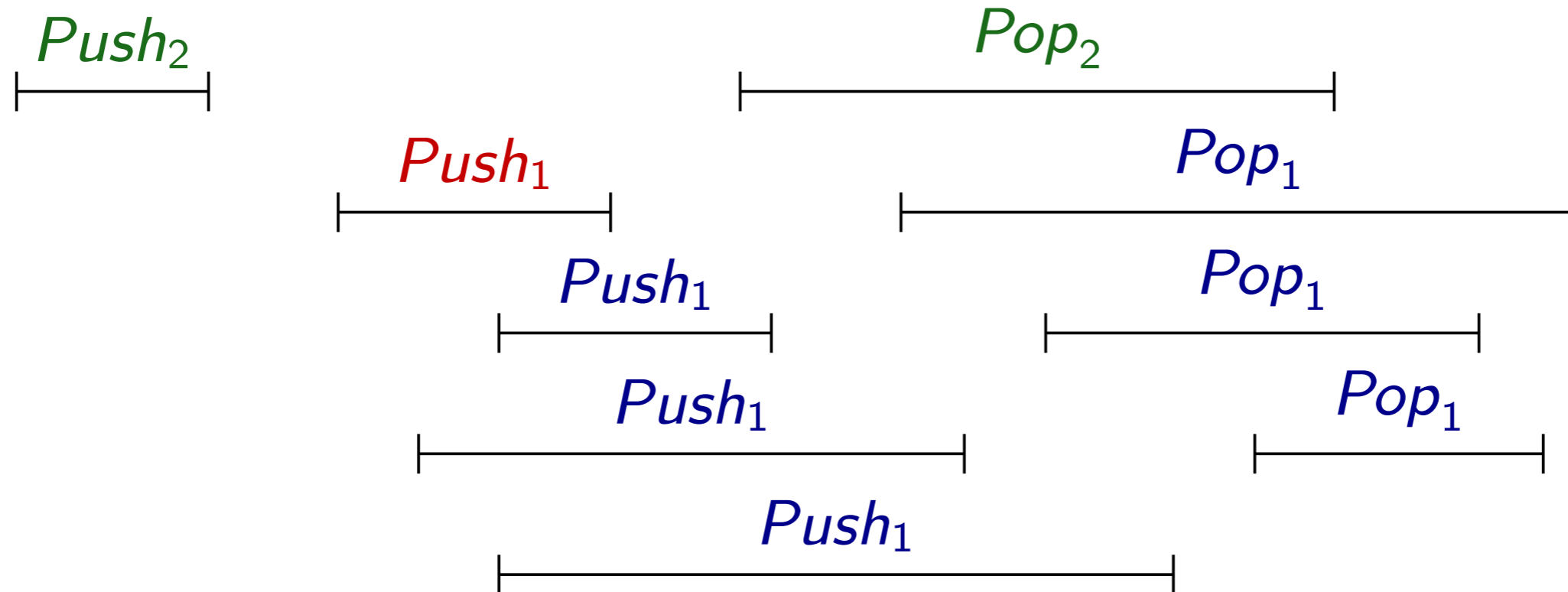


$\text{Enq}(1) < \text{Enq}(2) \ \& \ \text{Deq}(2) < \text{Deq}(1)$

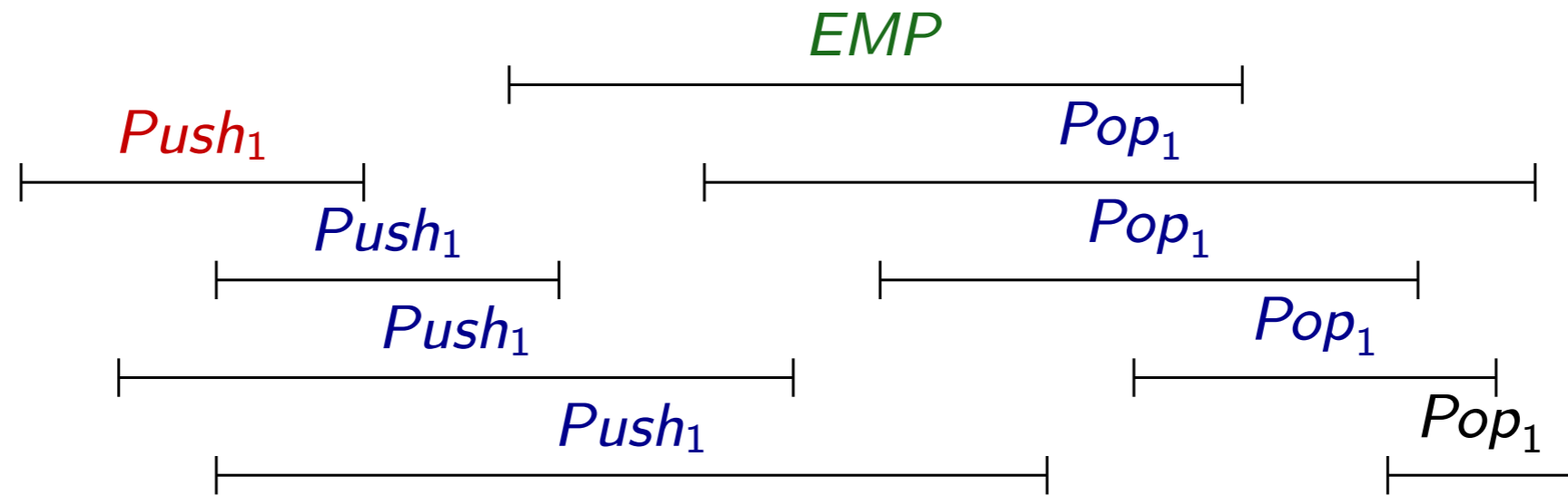
# Empty Violation



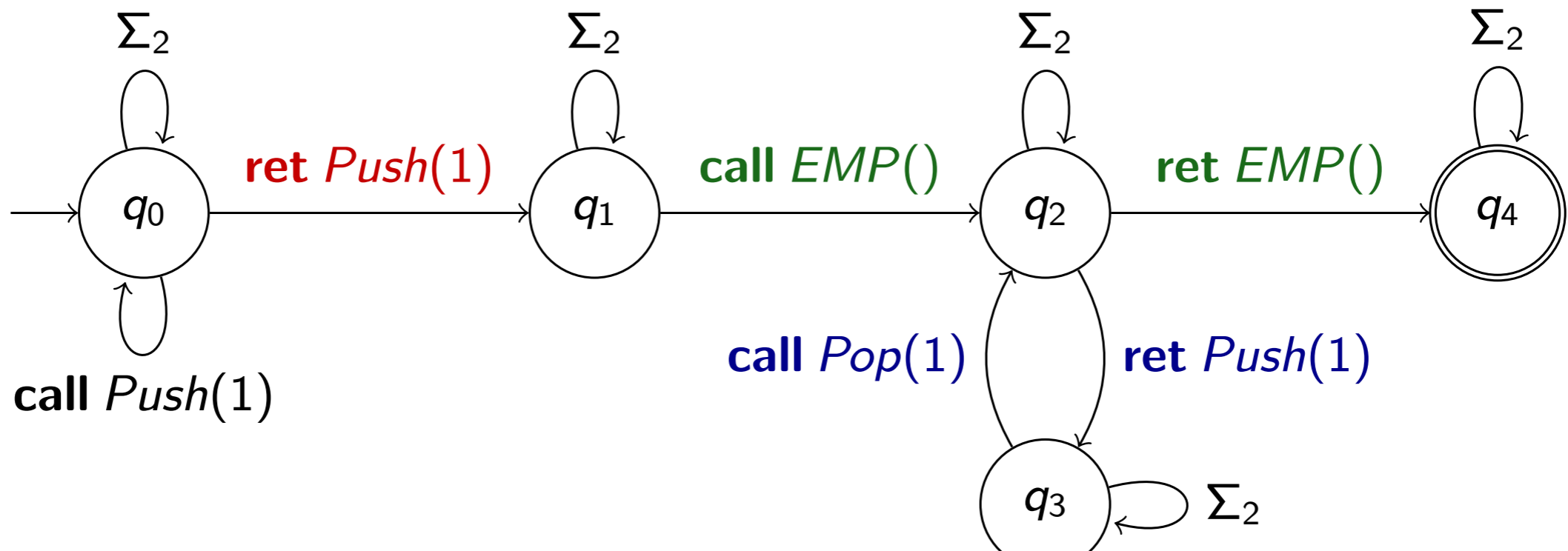
# Order Violation cont.



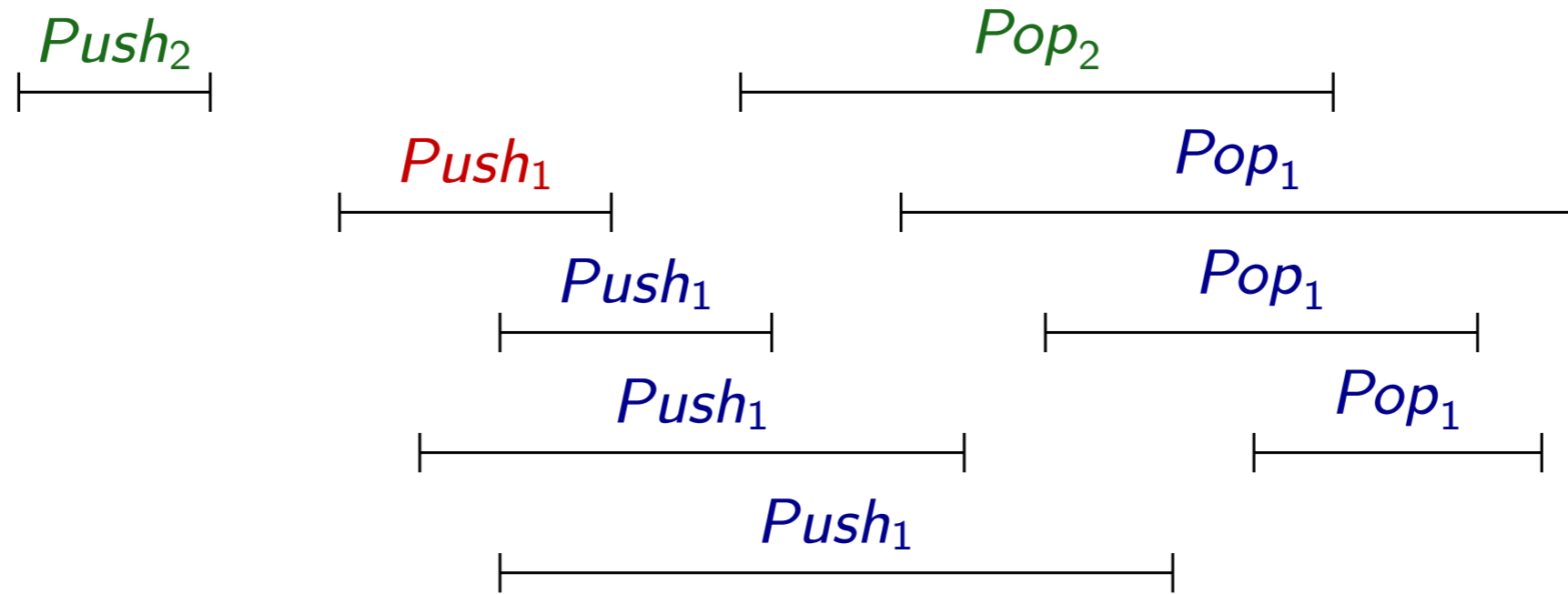
# Automaton for Empty Violation



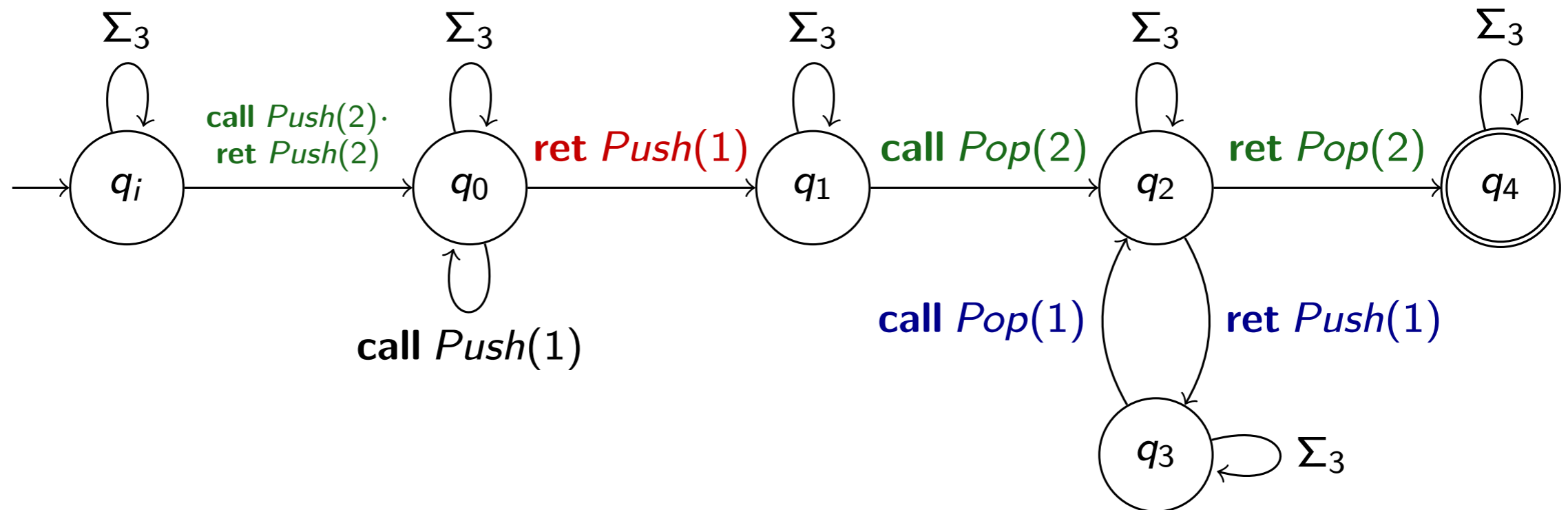
Recognized by:



# Automaton for Push-Pop Order Violation



Recognized by:





# Linearizability to State Reachability

Thm:

For each  $S$  in {Stack, Queue, Mutex, Register}, there is an automaton  $A(S)$  s.t. for every data independent concurrent object  $C$ ,  $C$  is linearisable wrt  $S$  iff the intersection of  $H(C)$  wrt  $H(A(S))$  is empty.

Same complexity as state reachability

# Conclusion/Future work

- Bounding concept based on the notion of interval-length
- OR checking  $\rightarrow$  Reachability problem, using counting
- Suitable bounding concept: low complexity, small bounds
- Application in Dynamic and Static Analysis
- $\Rightarrow$  Automatic synthesis of “specification”
  
- Reduction to SR for a common concurrent objects
- Reuse of existing verification technology
- $\Rightarrow$  Extension to other kind of objects, e.g., sets
- $\Rightarrow$  Abstractions / under-approximate analysis