Expressivity of Non-atomic Asychronous Networks

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with Antoine Durand-Gasselin, Javier Esparza and Rupak Majumdar

processes have no identity

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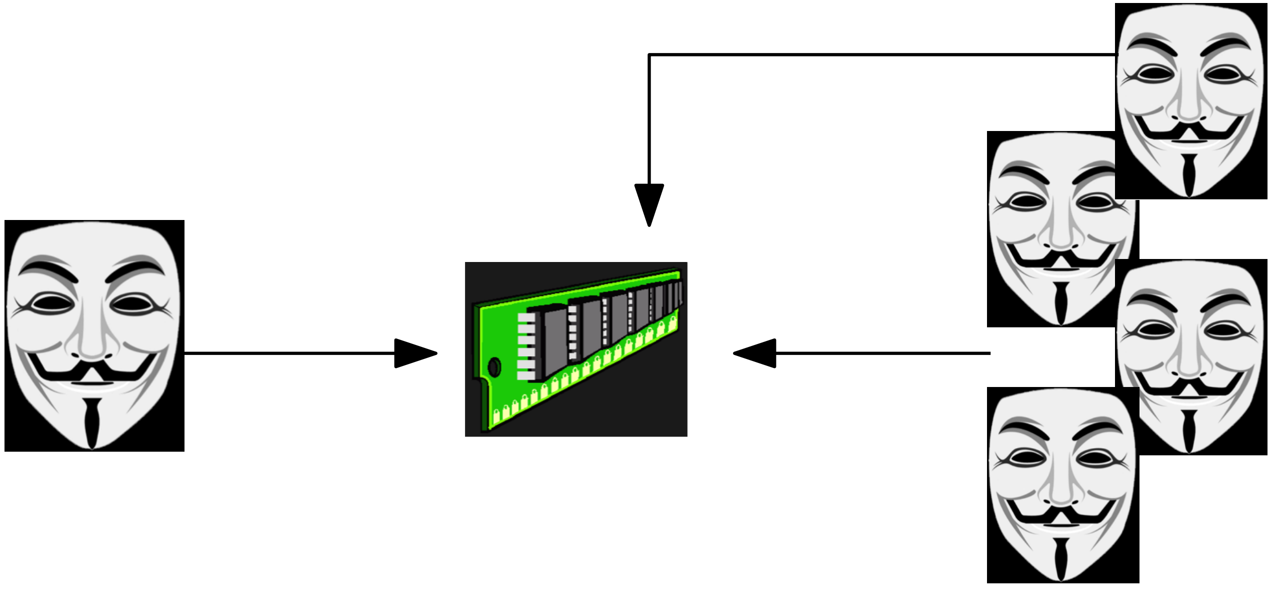
communication through a shared bounded-value register



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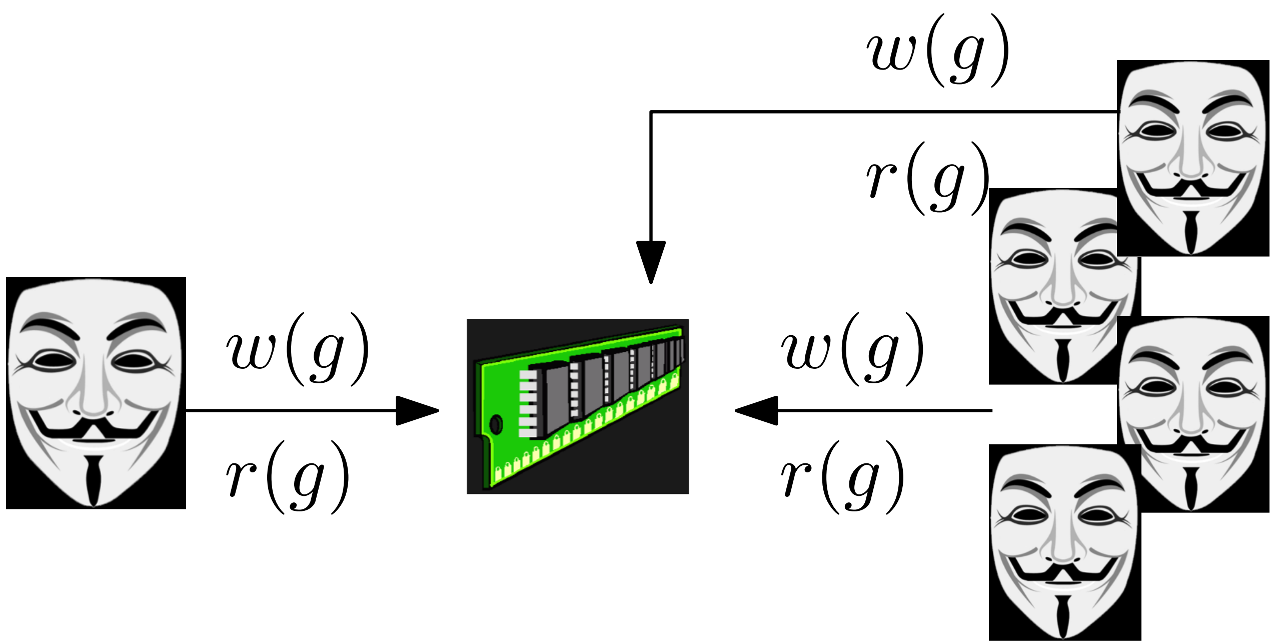
register can be read from, written to but not **locked**



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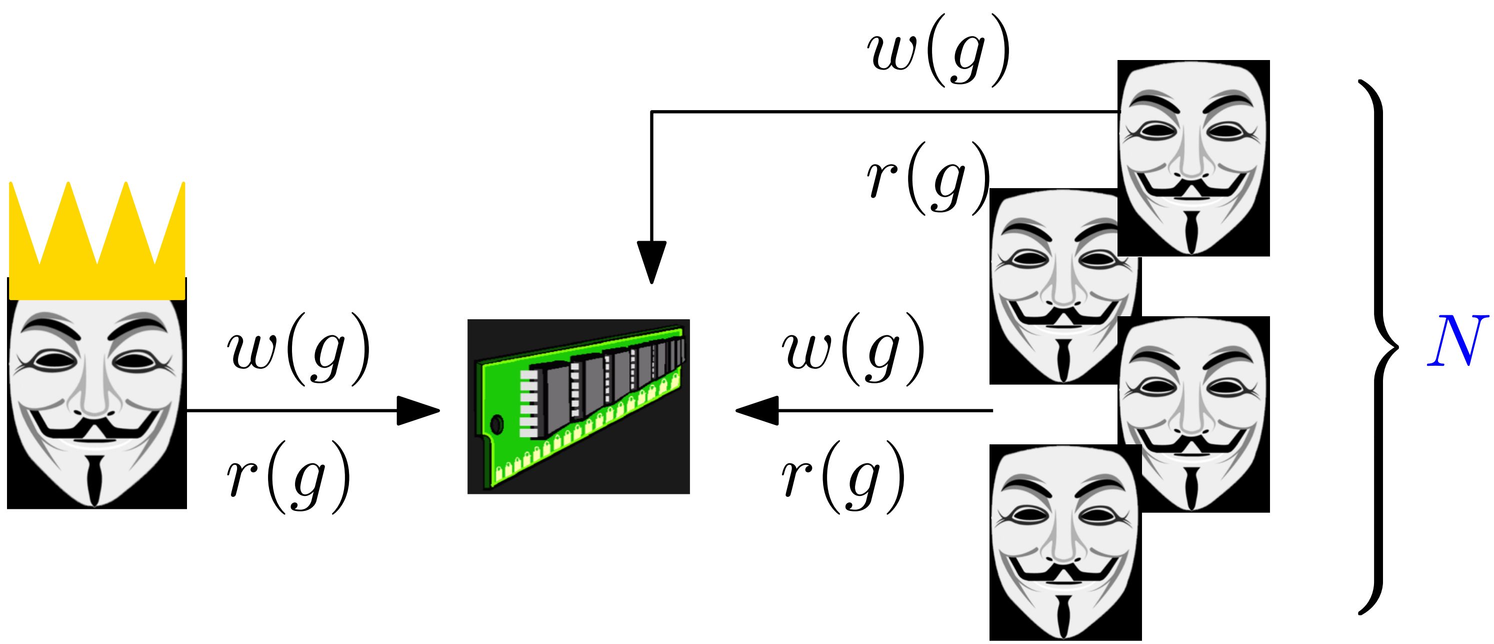

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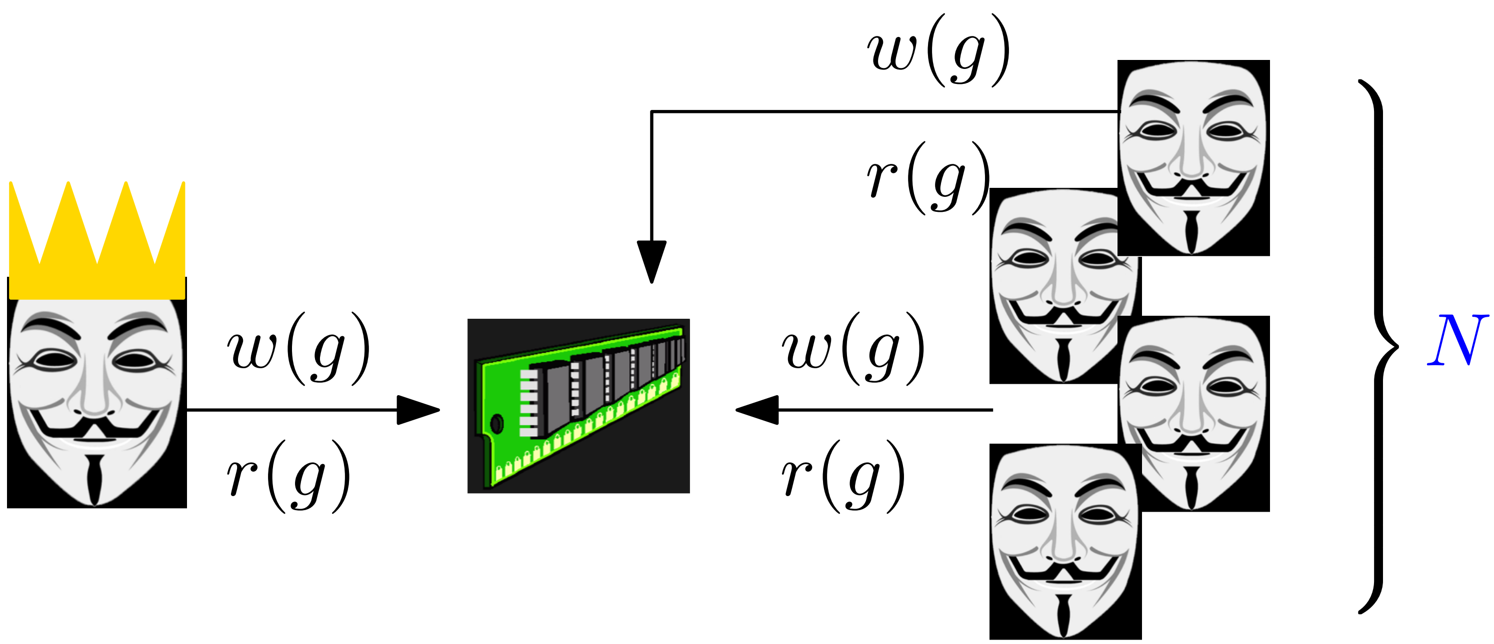
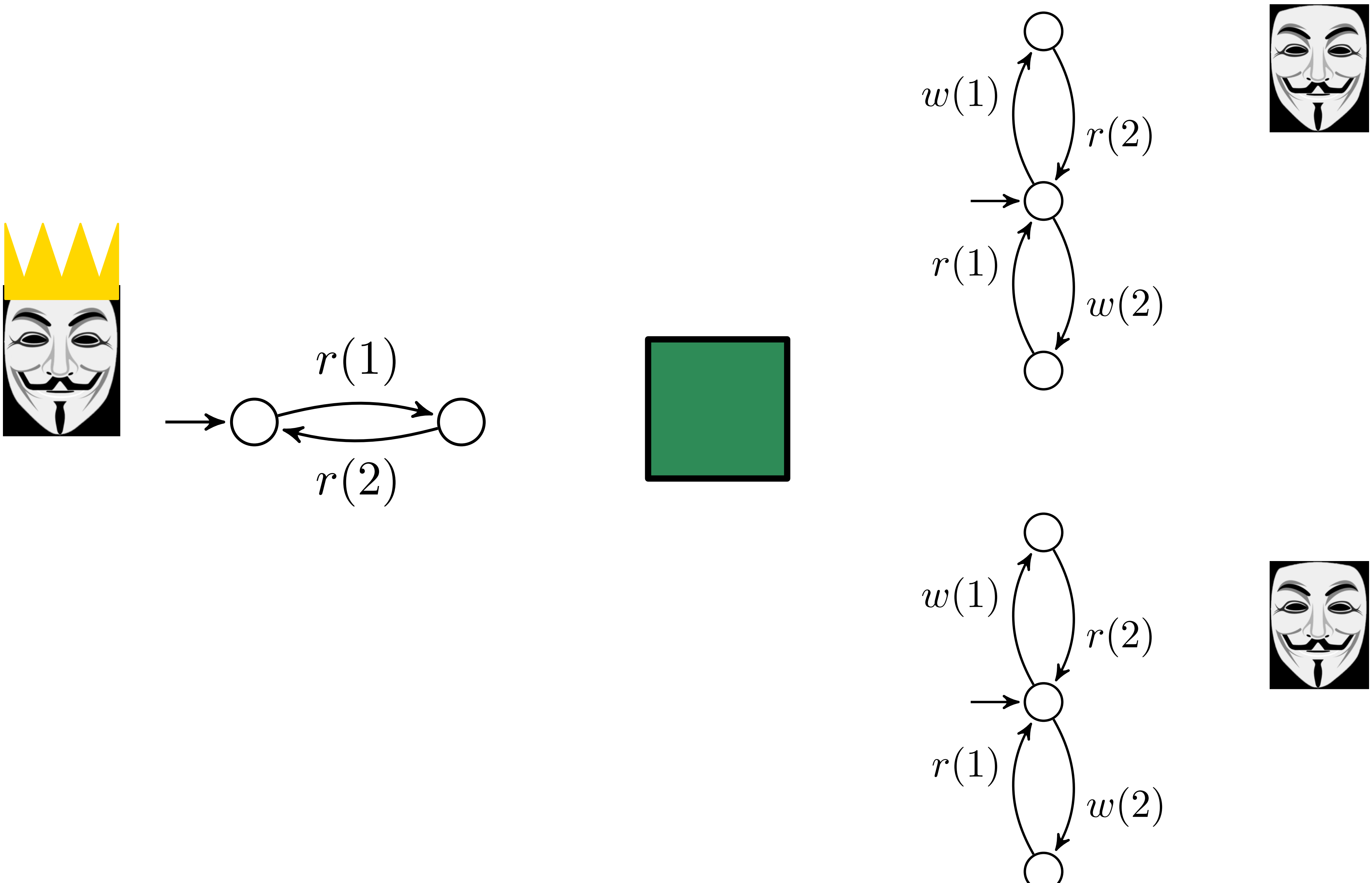


What can such network compute?


$r(1)$
$r(2)$
$w(1)$
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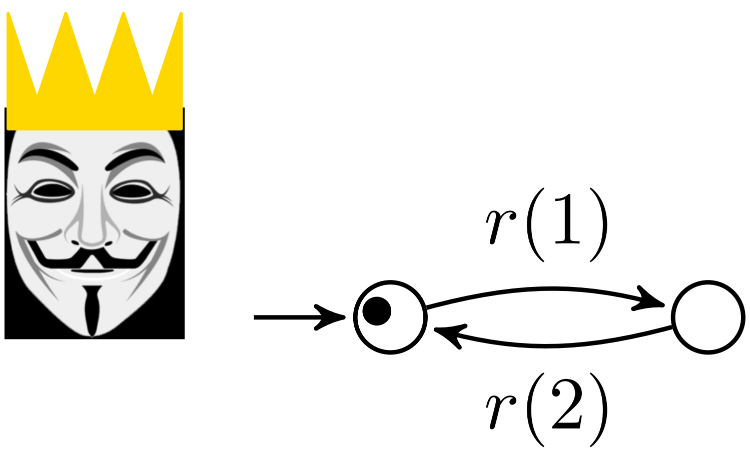
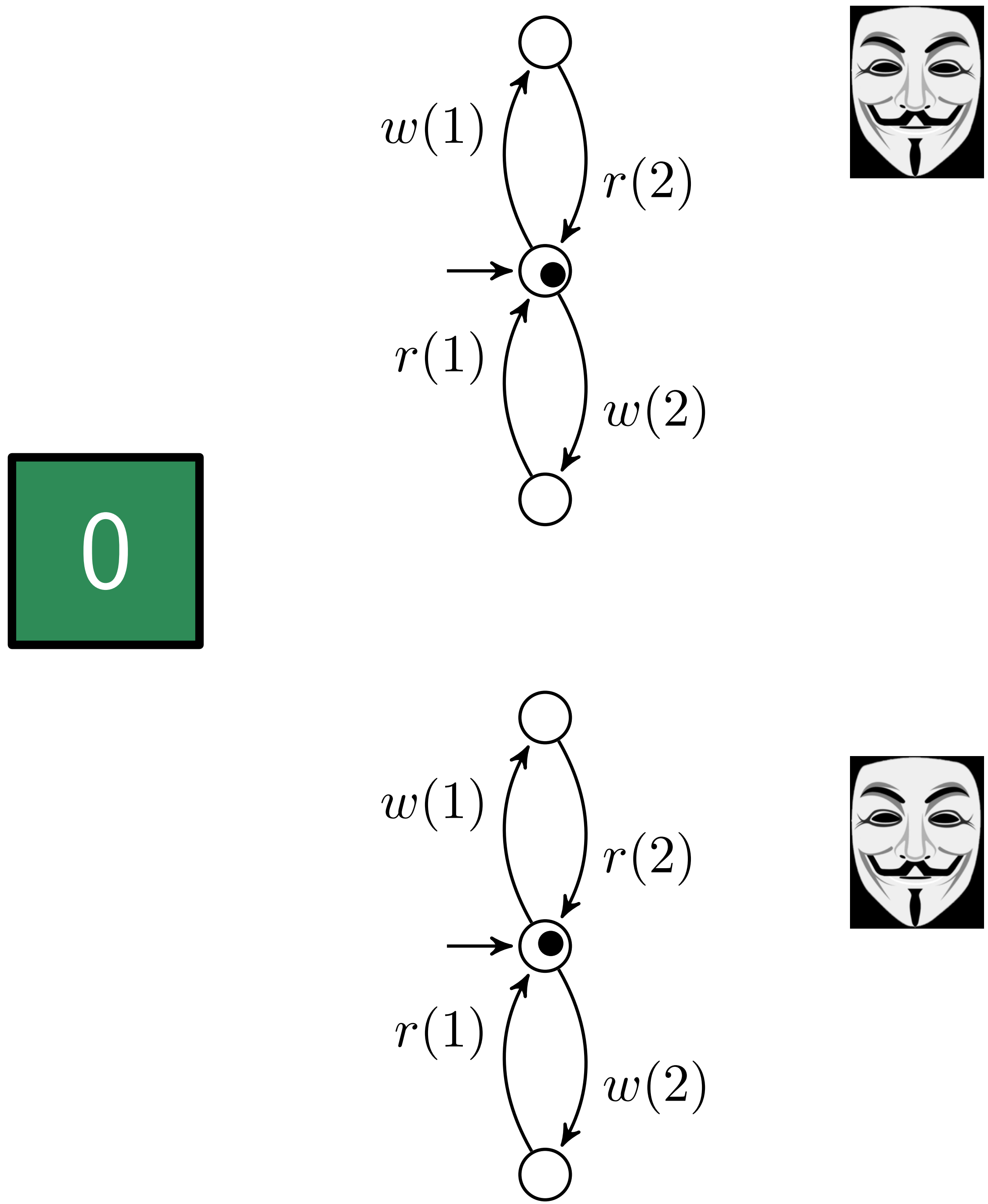
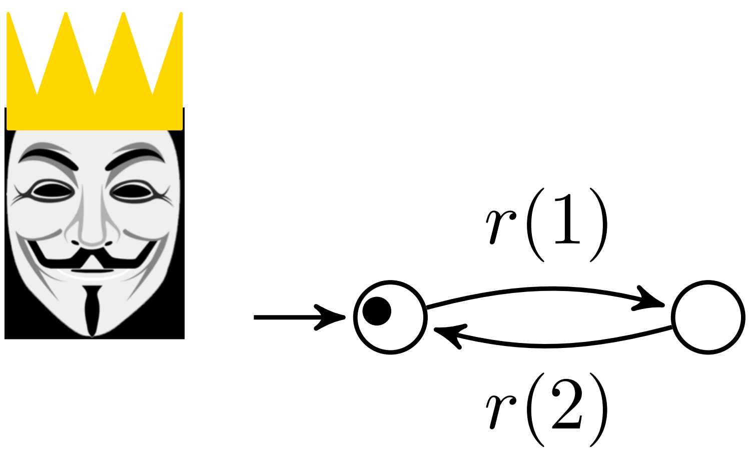
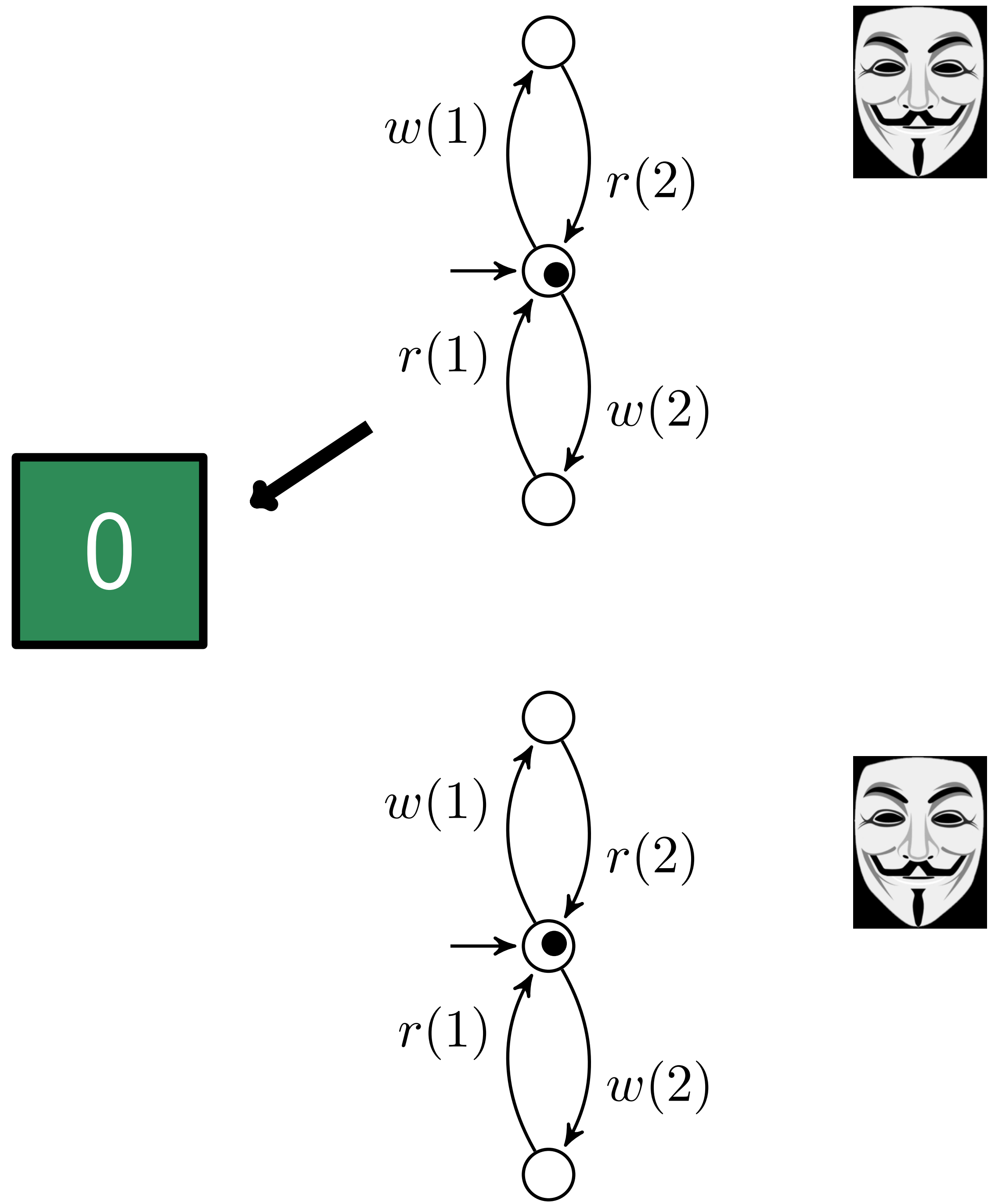




Run:





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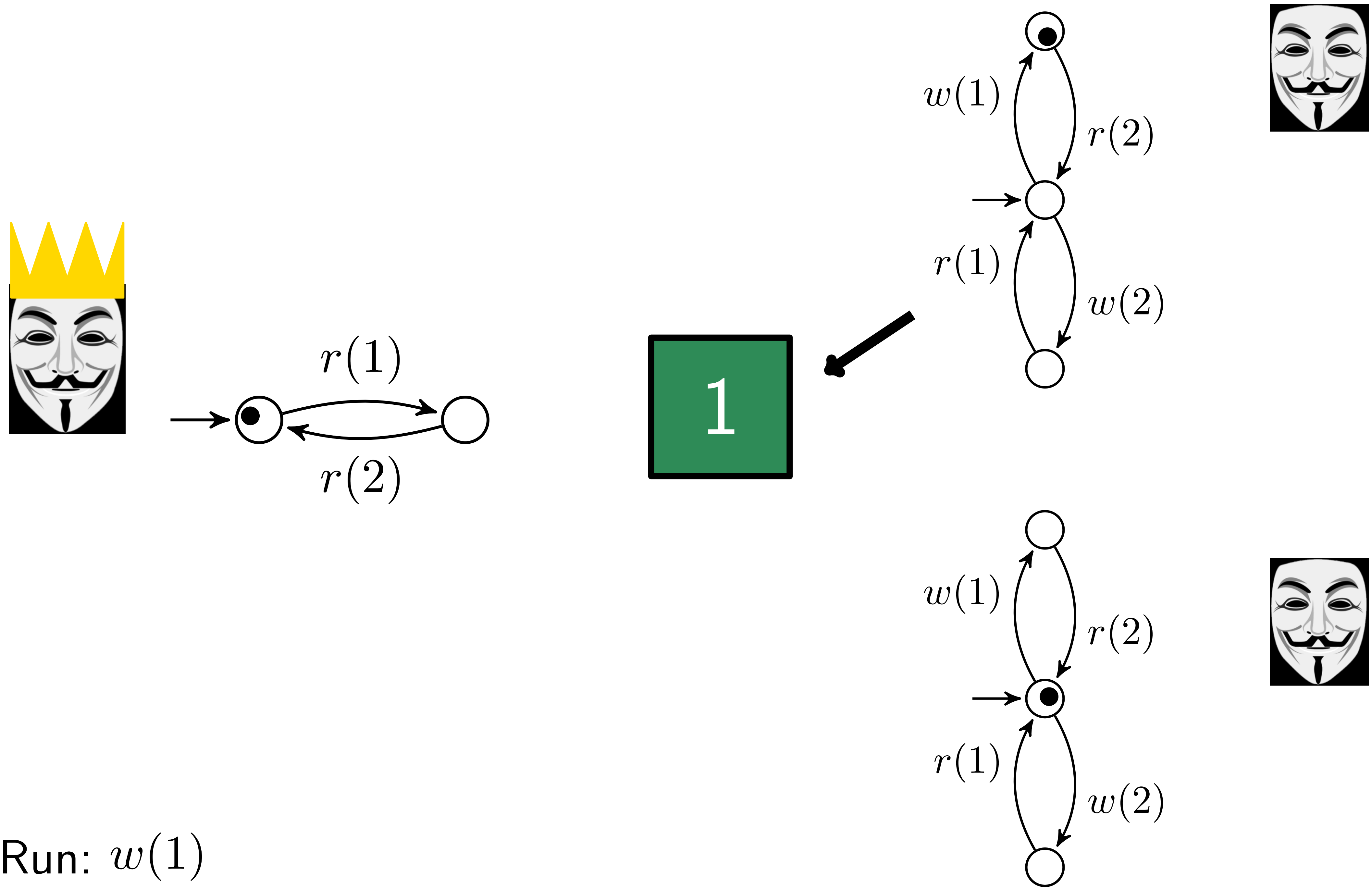
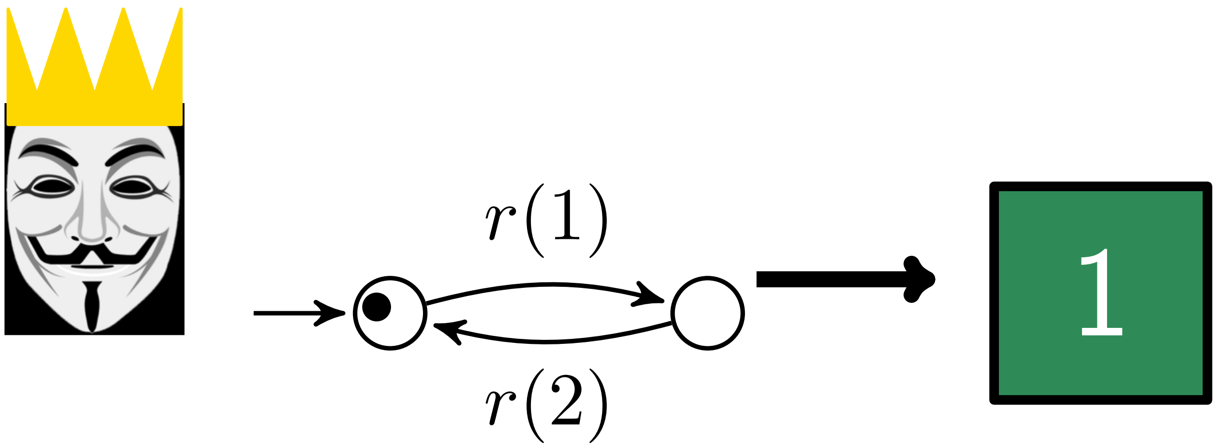
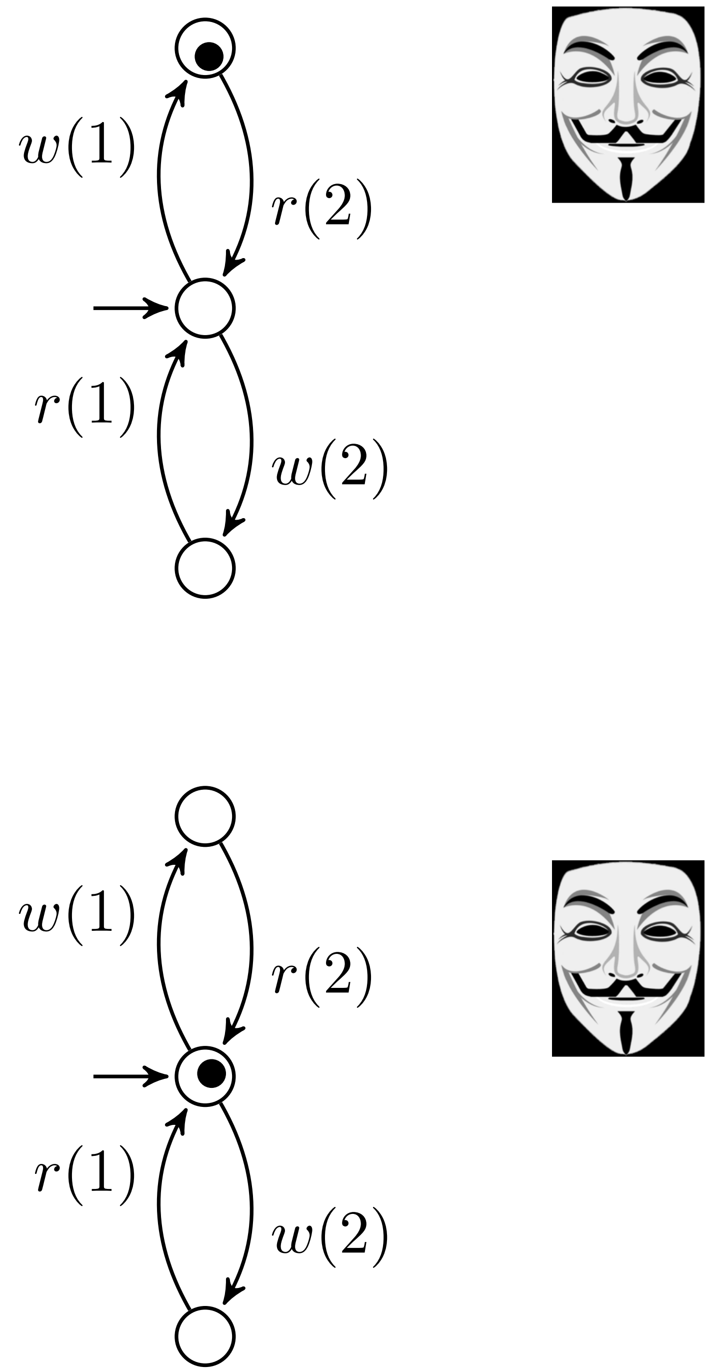


Run: $w(1)$





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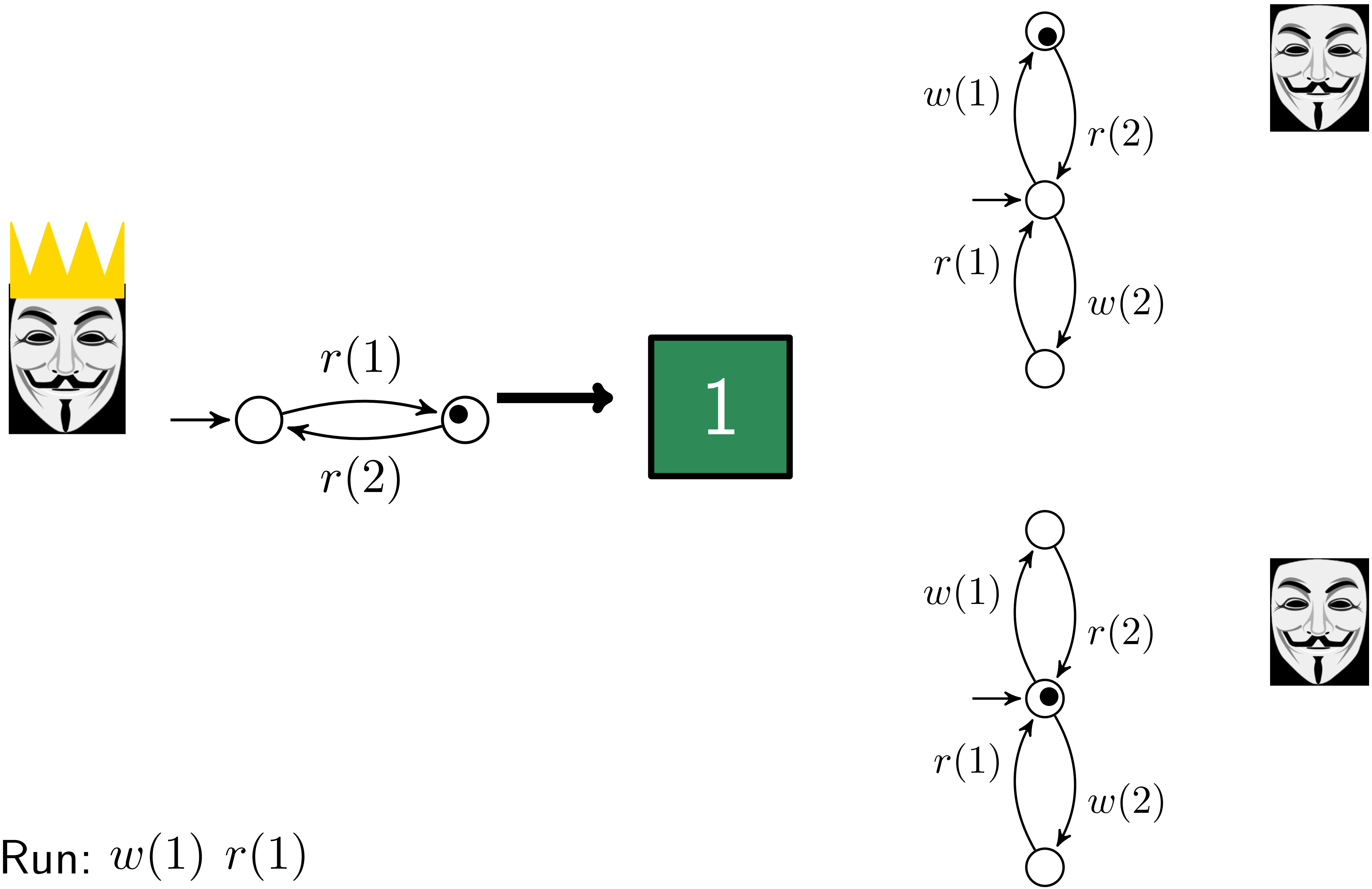
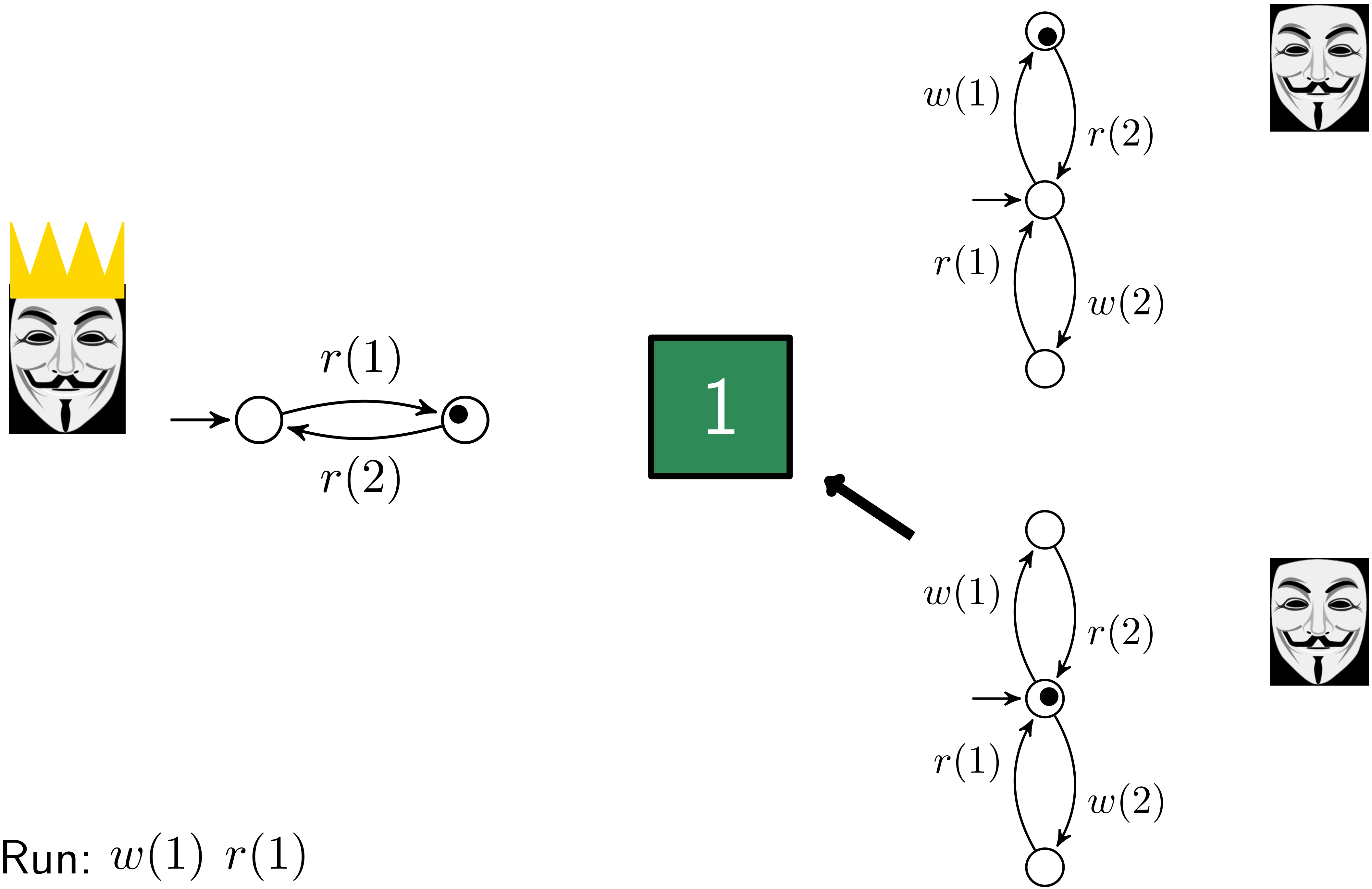


Run: $w(1)\ r(1)$



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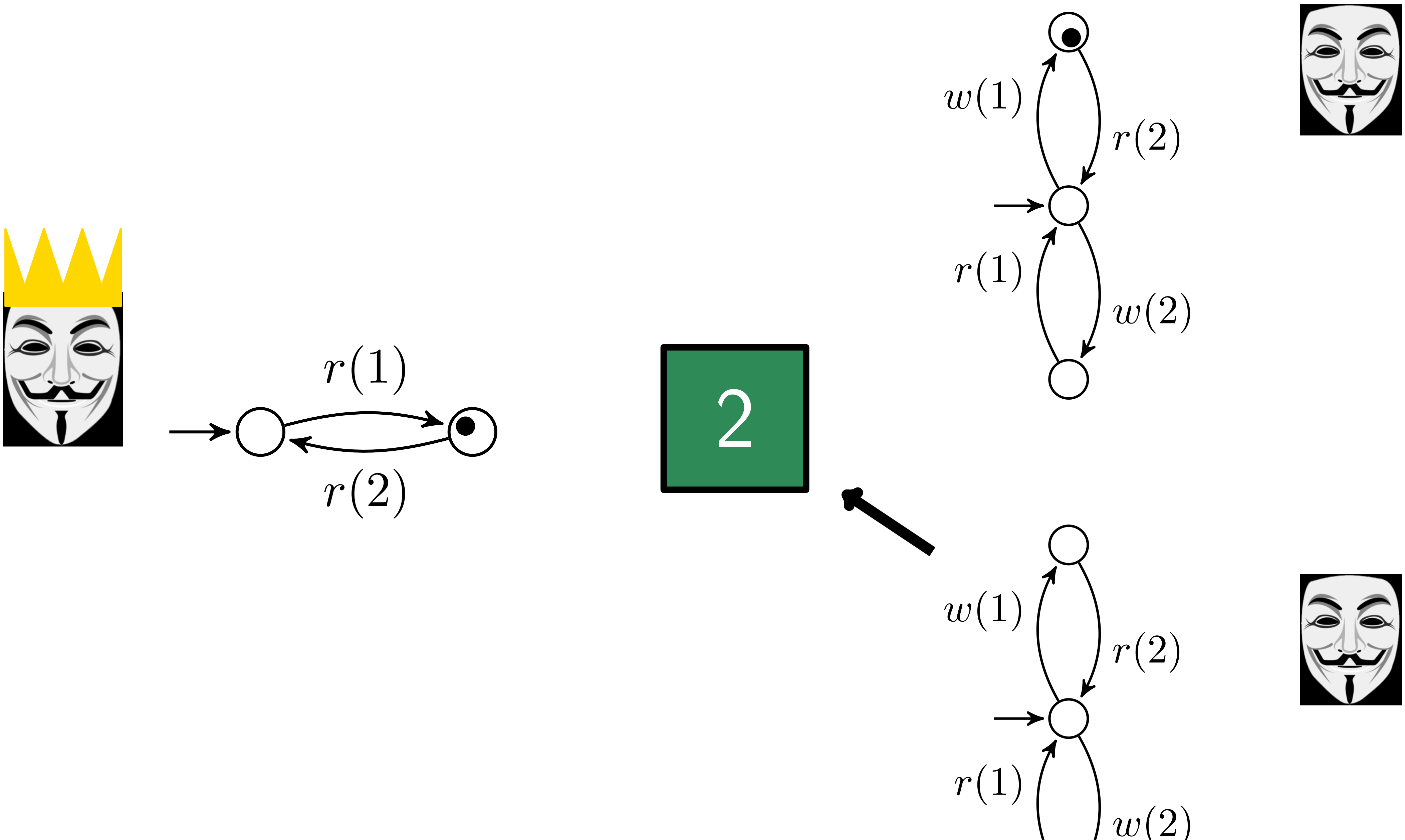
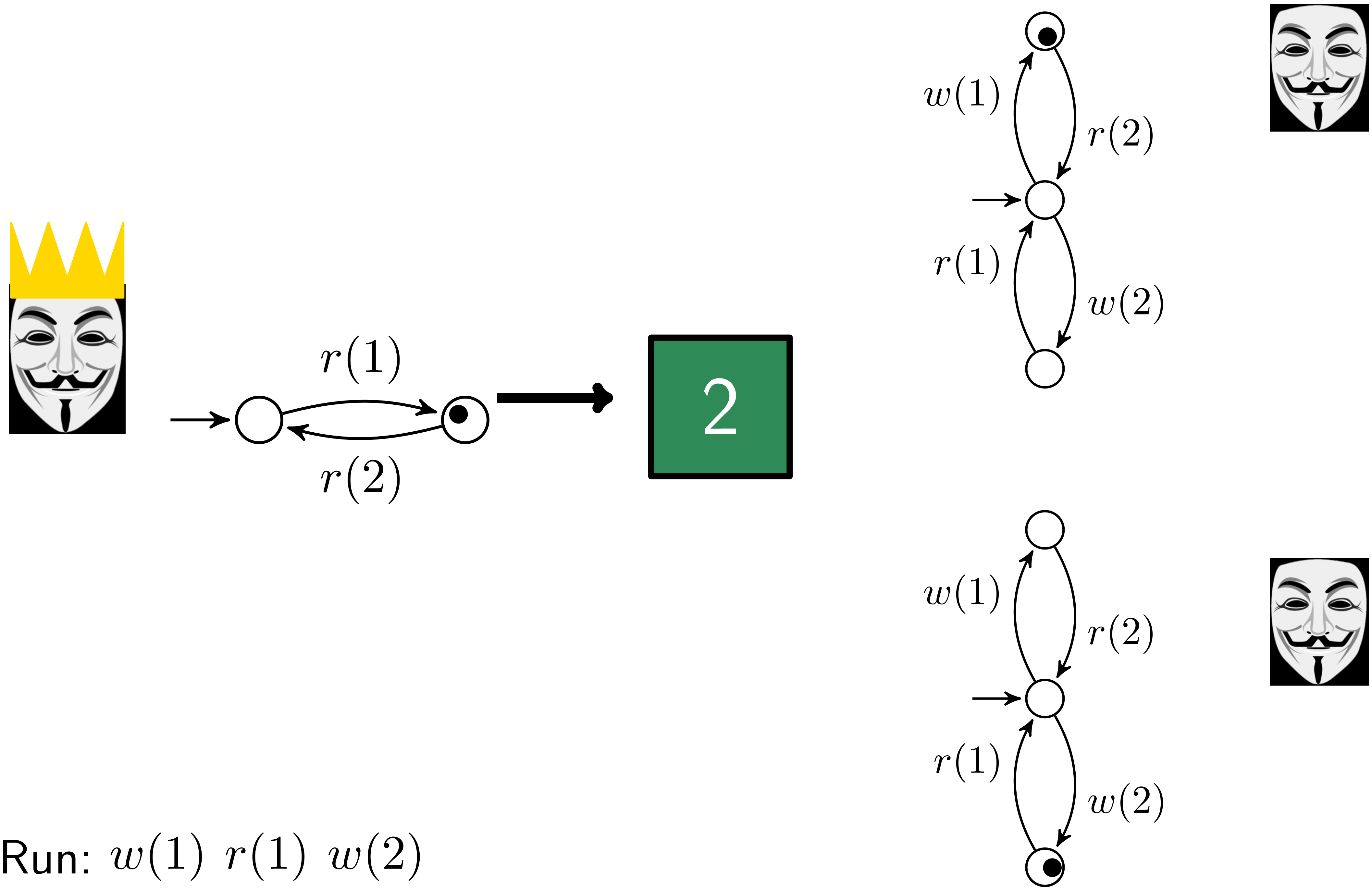


Run: $w(1)\ r(1)\ w(2)$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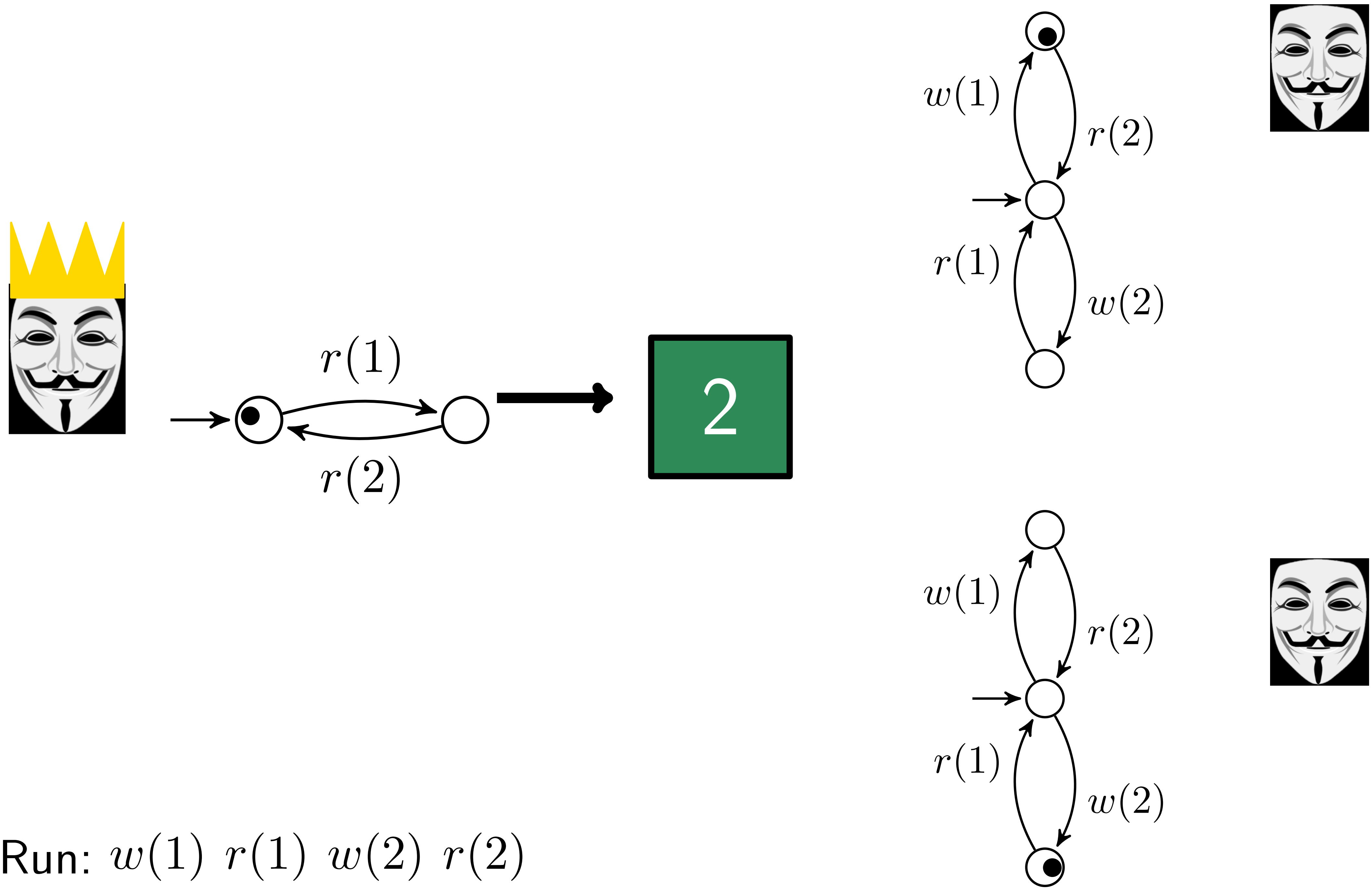

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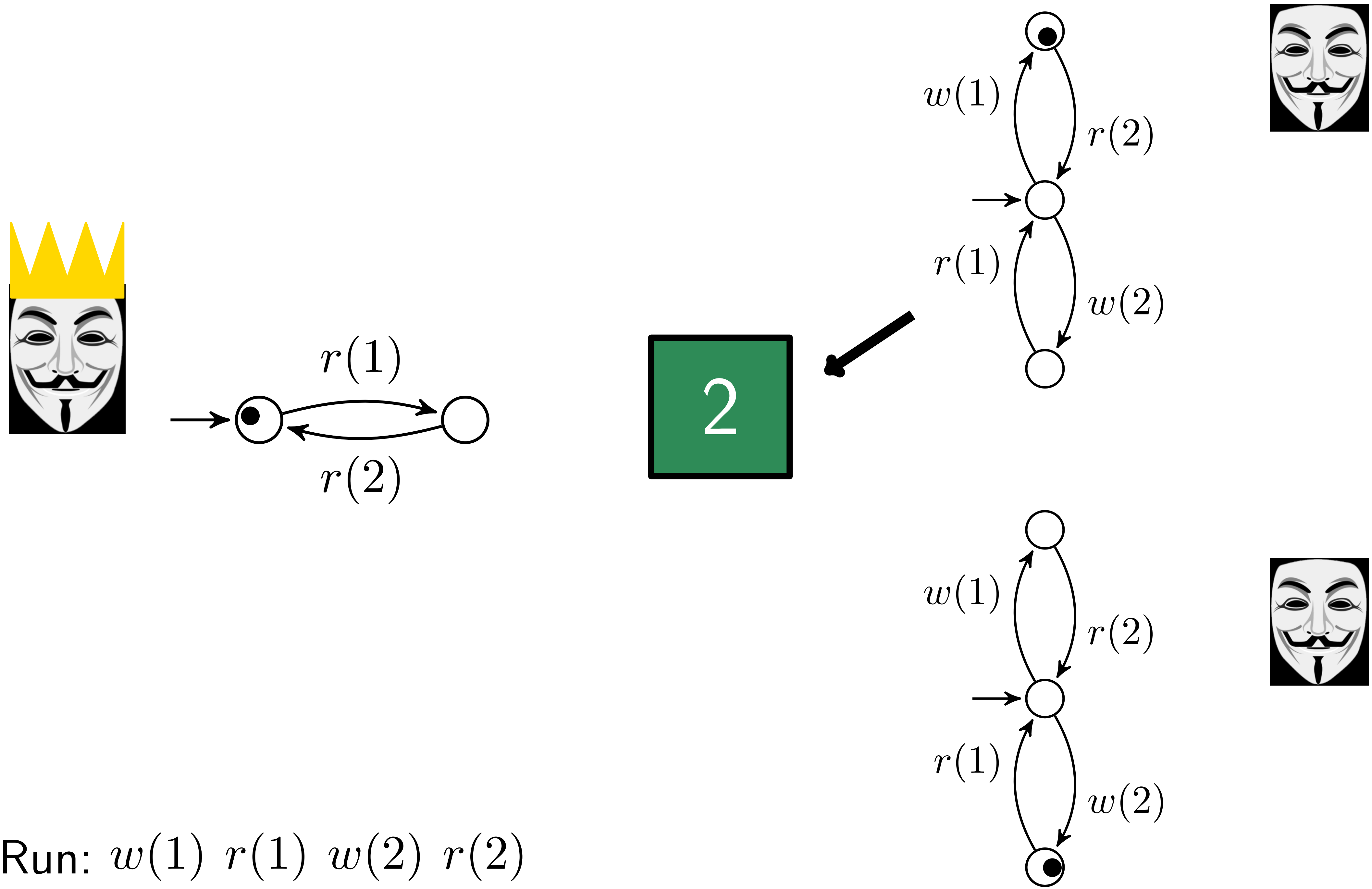

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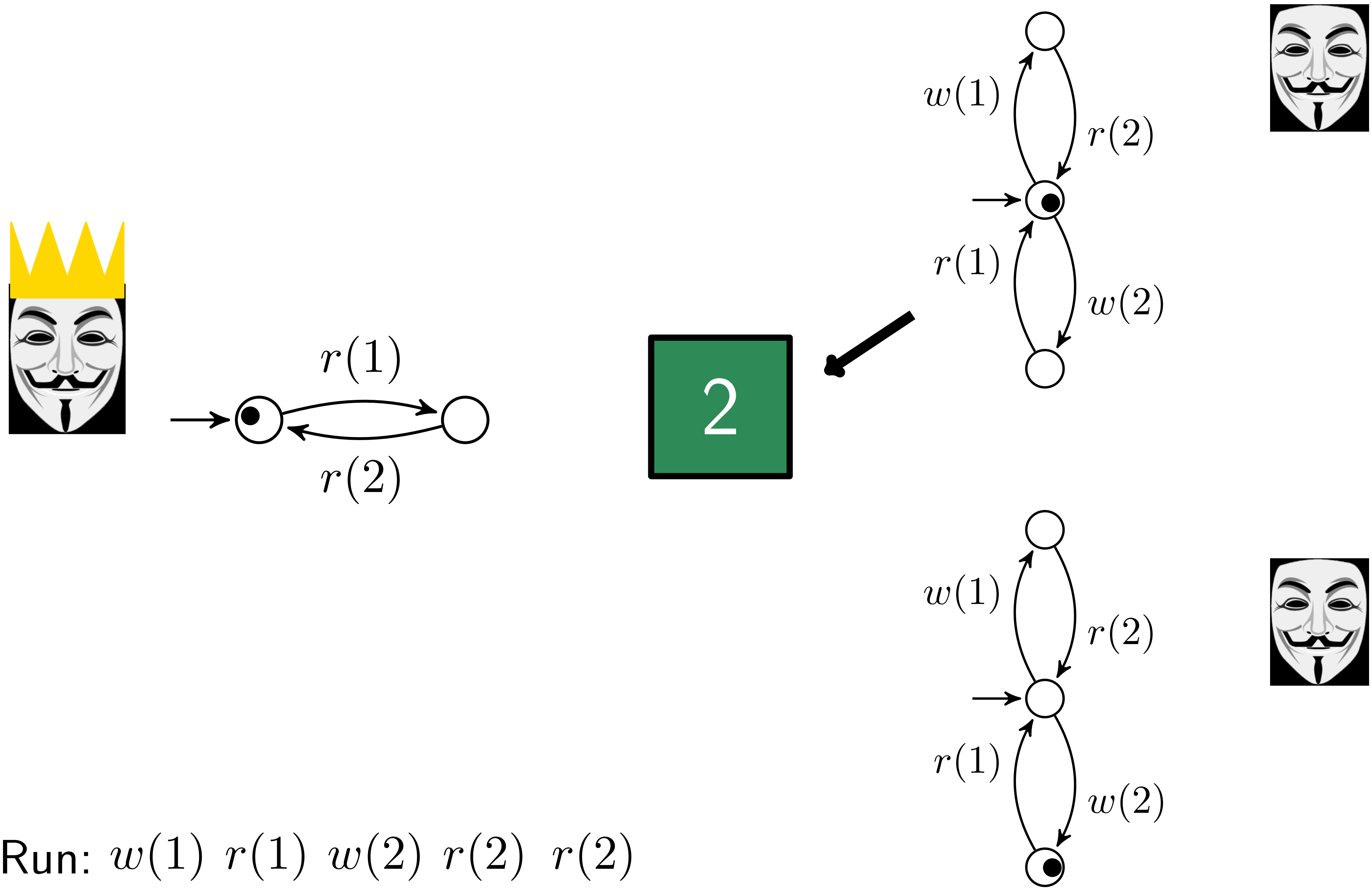

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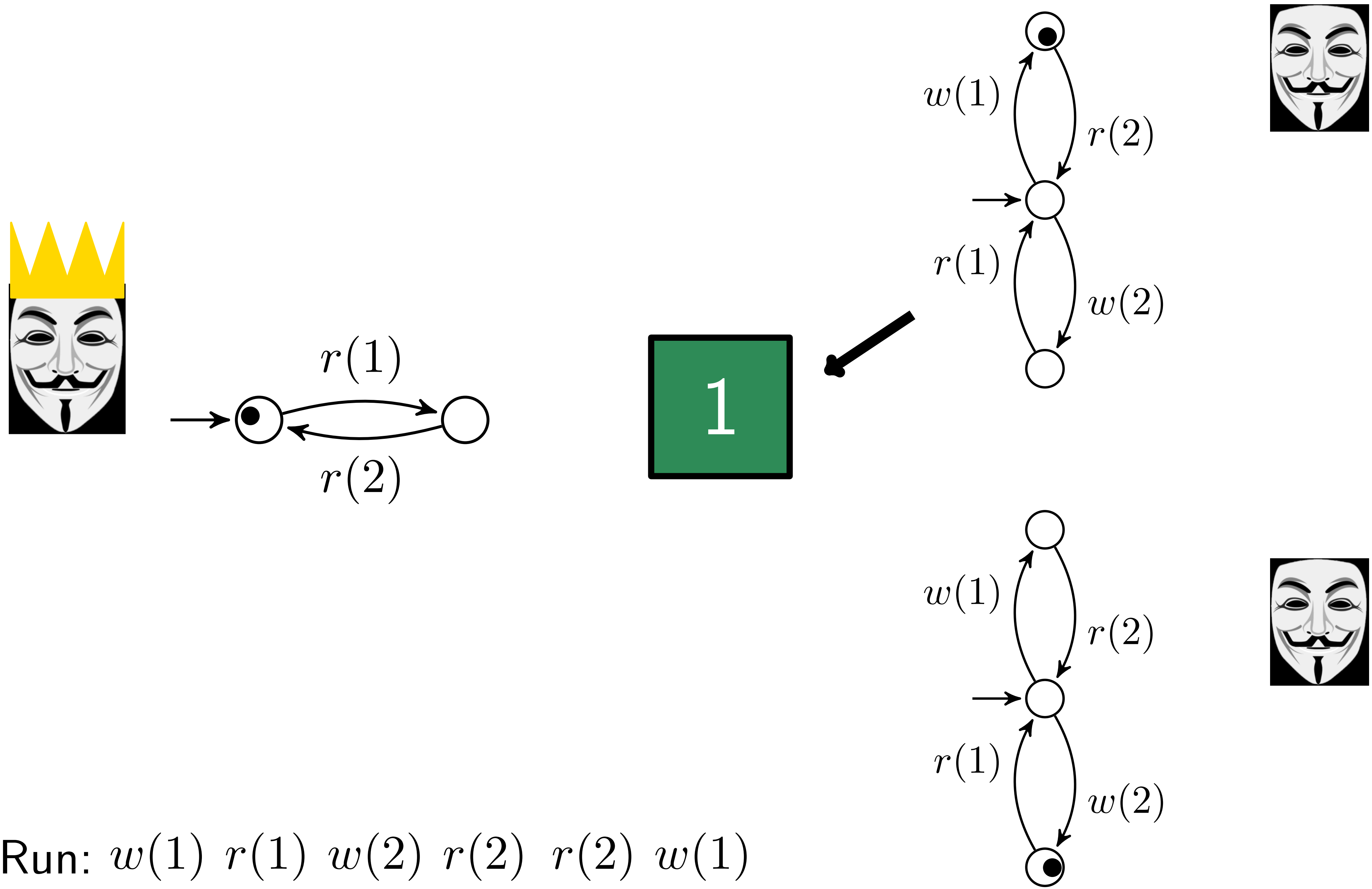

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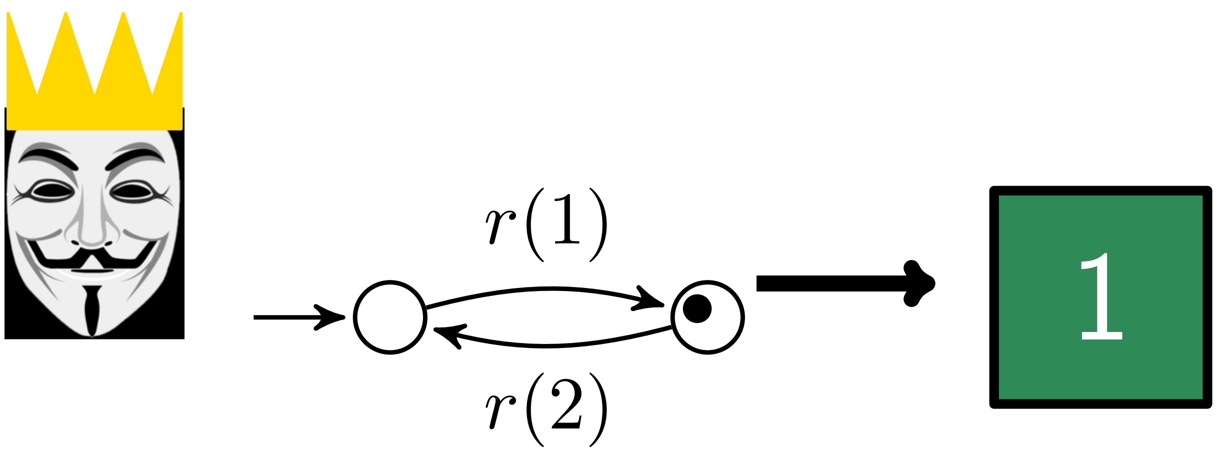

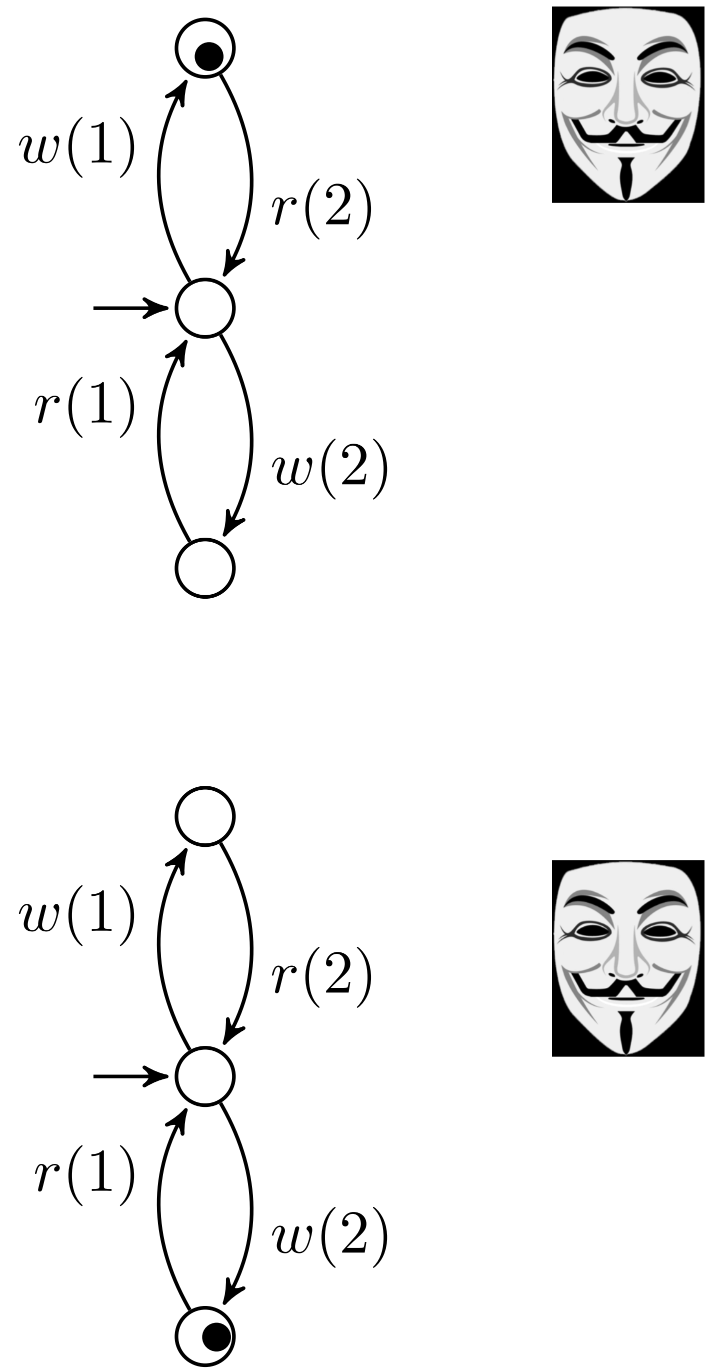

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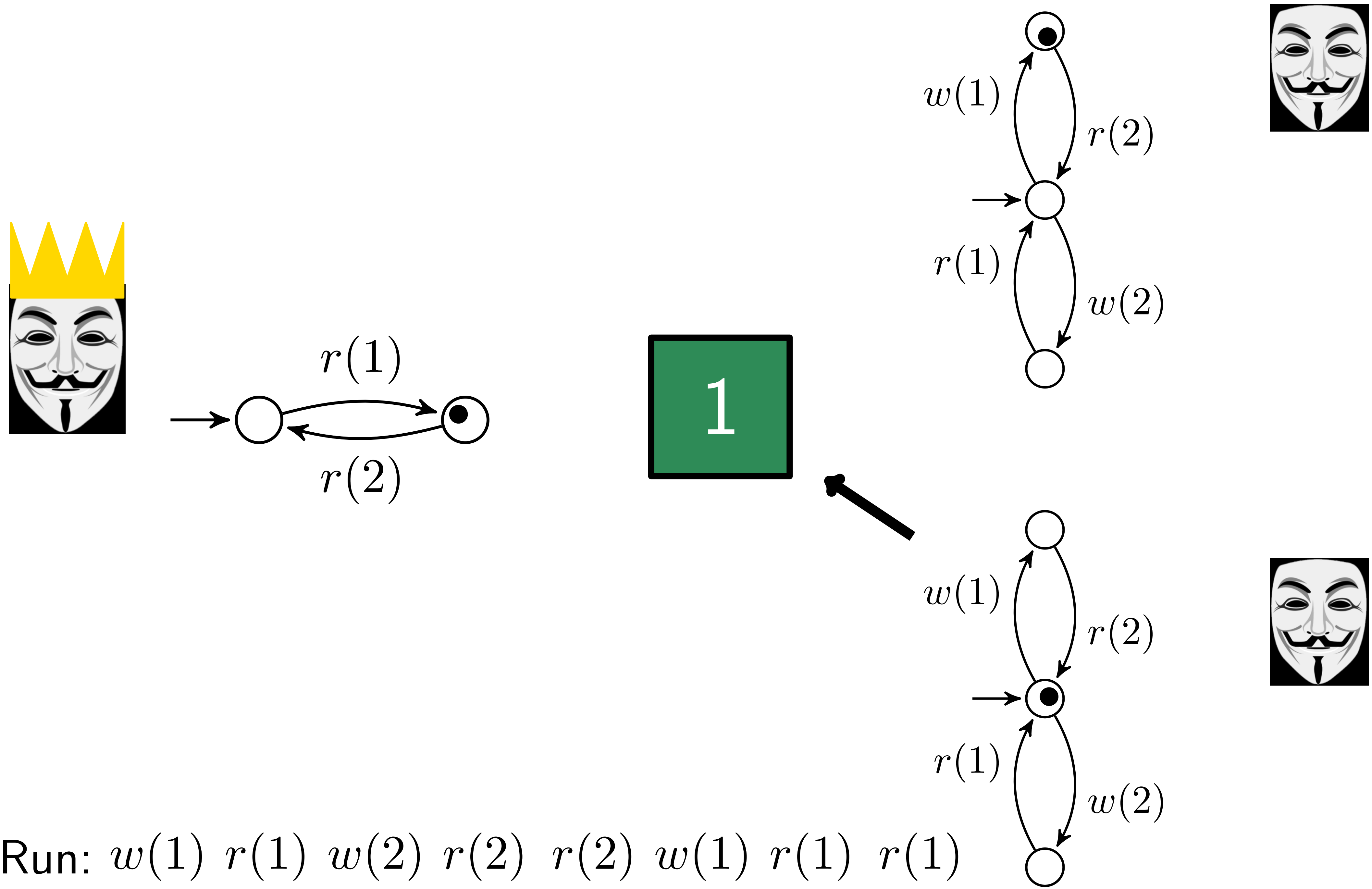

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How much can non-atomic networks compute?

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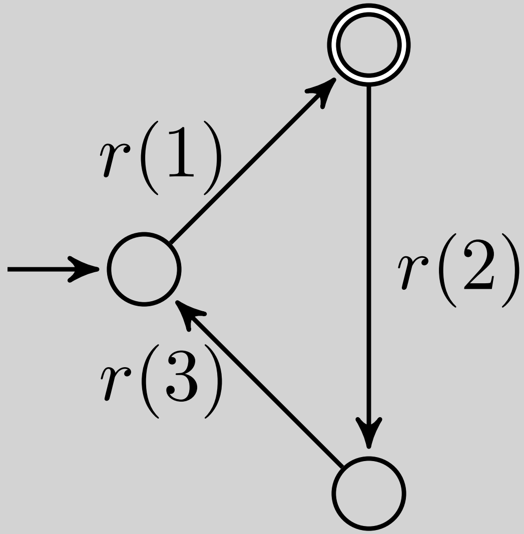
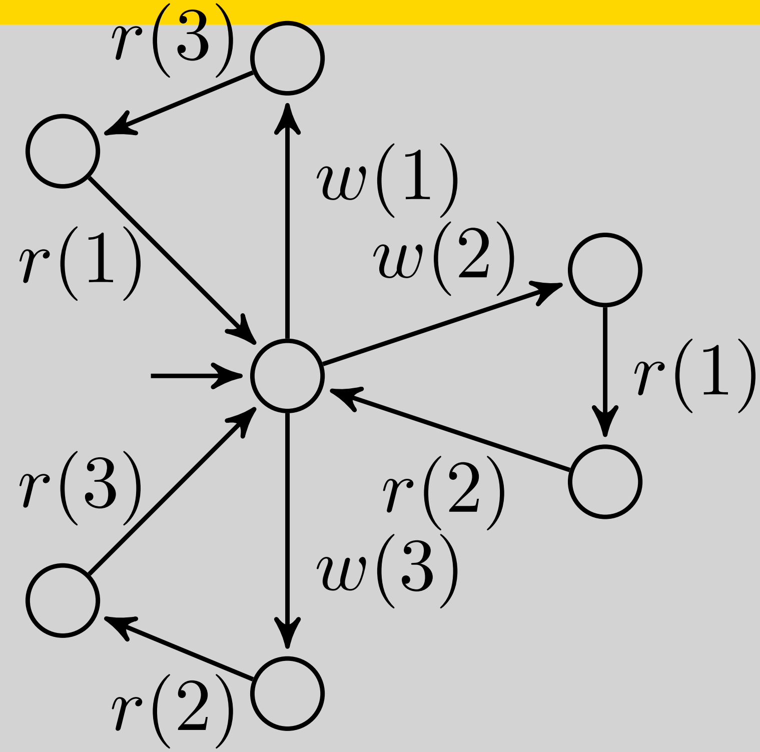
Today solving means repeatedly reaching q

Parameterized model-checking

Given:





N copies

Is there N:   have an accepting run ?

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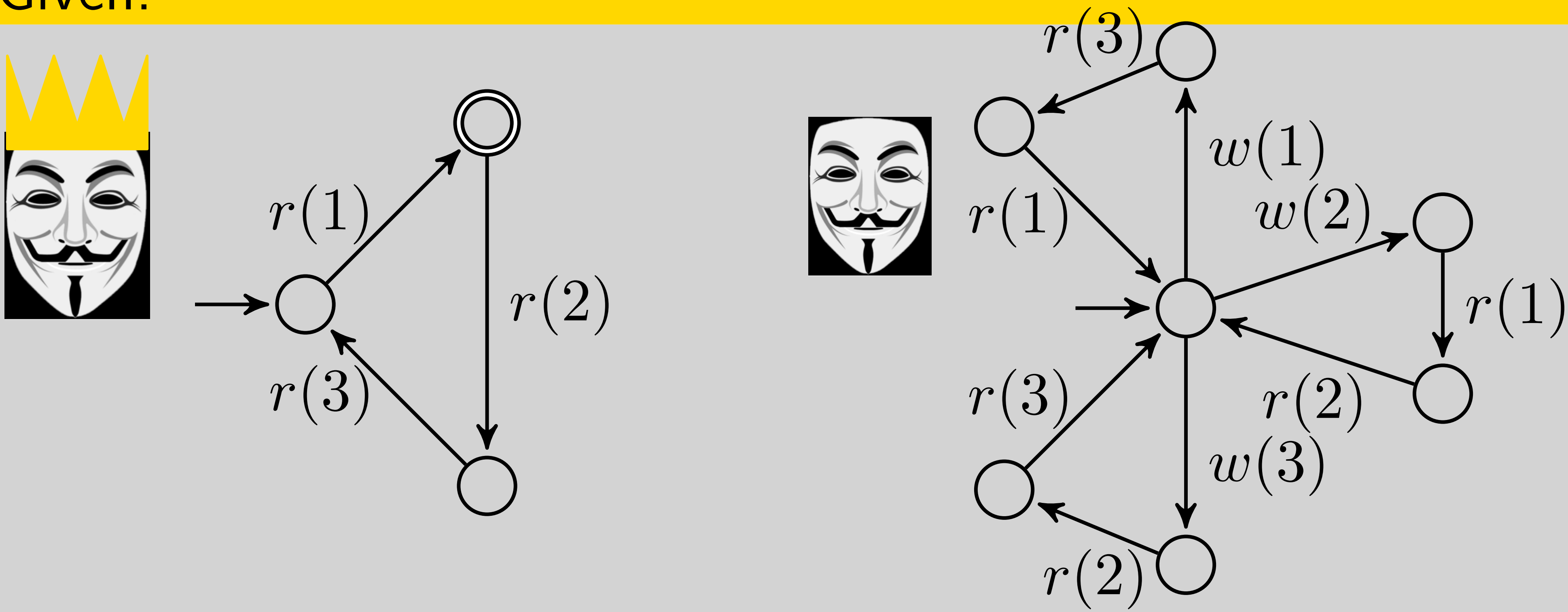
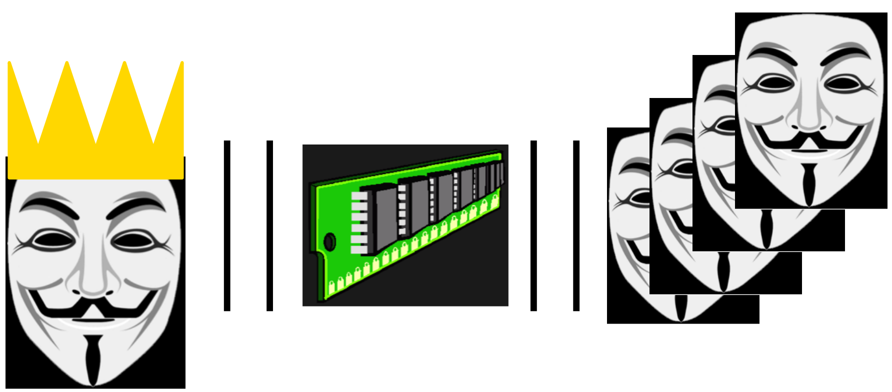


N copies

Is there N:  have an accepting run ?

Is there an infinite accepting run with finitely many processes?

NP-complete

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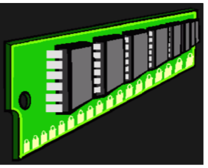
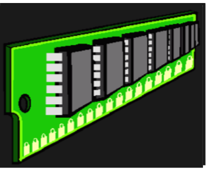
when all processes are given by finite state machine



$w(1)$ $r(1)$ $r(1)$ $w(2)$ $r(2)$ $w(1)$ $r(1)$ $r(1)$ $w(3)$ $r(3)$



$r(1)$ $w(1)$

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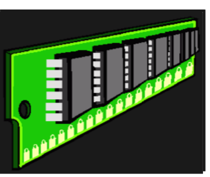 $w(1)$ $r(1)$ $r(1)$ $w(2)$ $r(2)$ $w(1)$ $r(1)$ $r(1)$ $w(3)$ $r(3)$

Copycat Lemma

$w(1)$ $r(2)$ $r(1)$ $r(3)$

$r(1)$ $w(2)$ $r(1)$ $w(3)$

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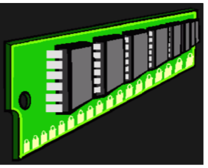

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	$w(1)$				$r(2)$		$r(1)$			$r(3)$
		$r(1)$		$w(2)$				$r(1)$	$w(3)$	
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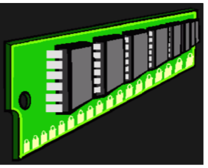	$w(1)$	$r(1)$	$r(1)$	$w(2)$	$r(2)$	$w(1)$	$r(1)$	$r(1)$	$w(3)$	$r(3)$

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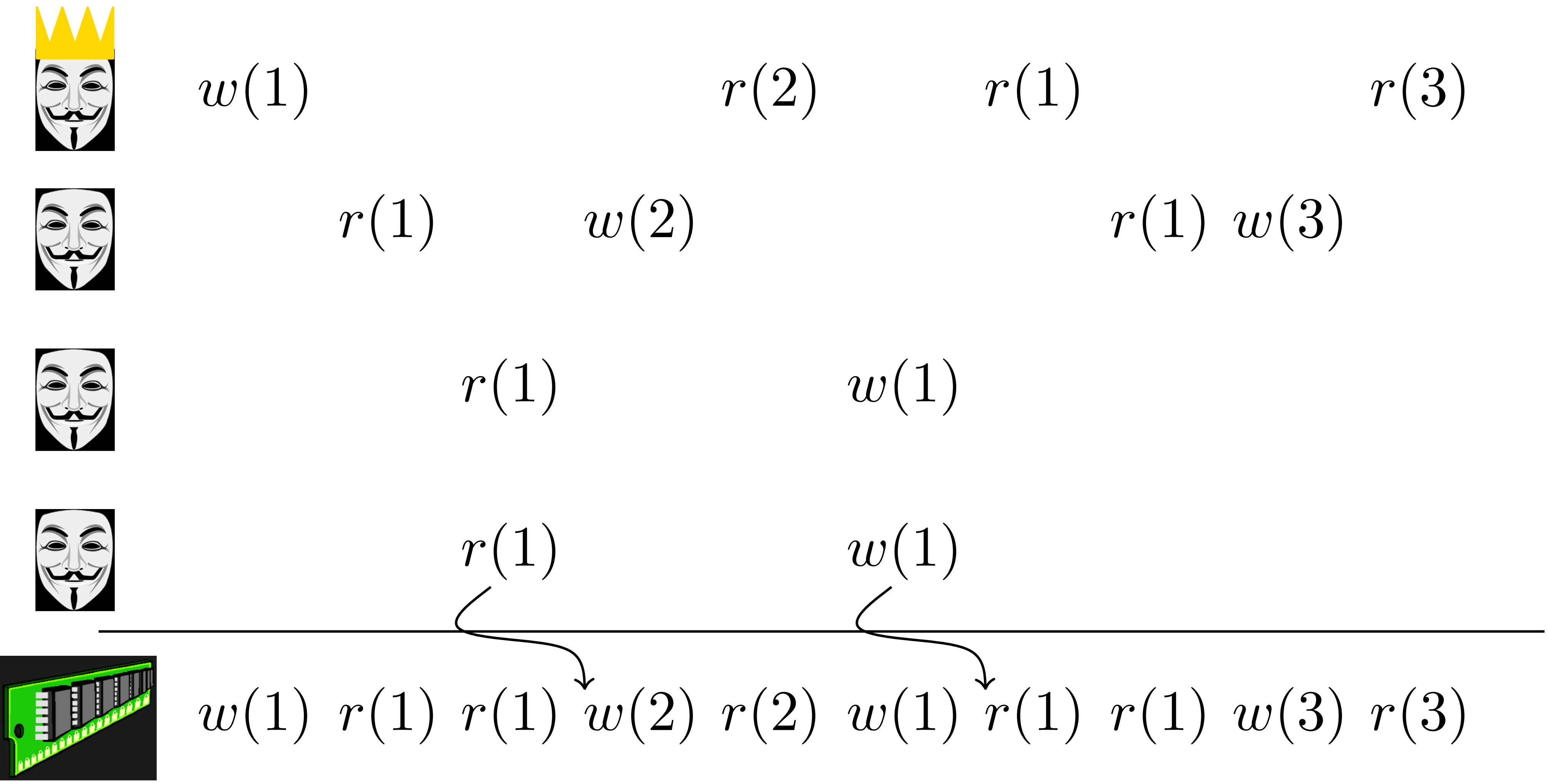

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 $r(1)$ $w(1)$

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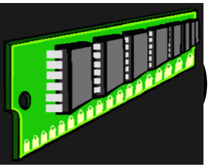 $r(1)$ $r(1)$ $r(1)$ $w(2)$ $r(2)$ $w(1)$ $w(1)$ $r(1)$ $r(1)$ $w(3)$ $r(3)$

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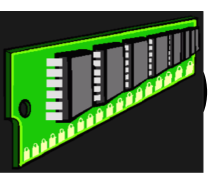
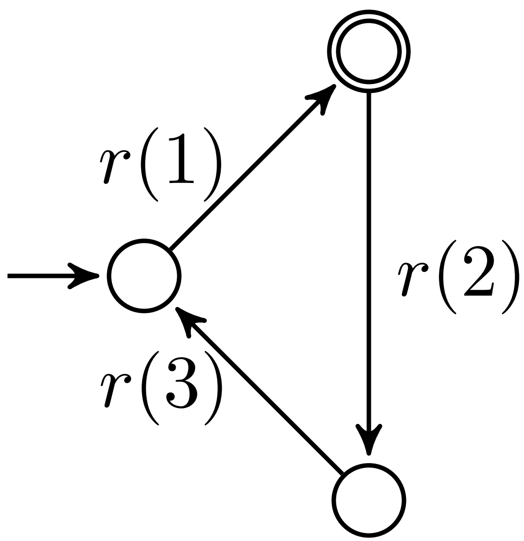
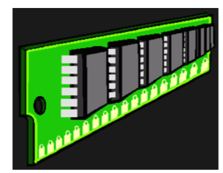
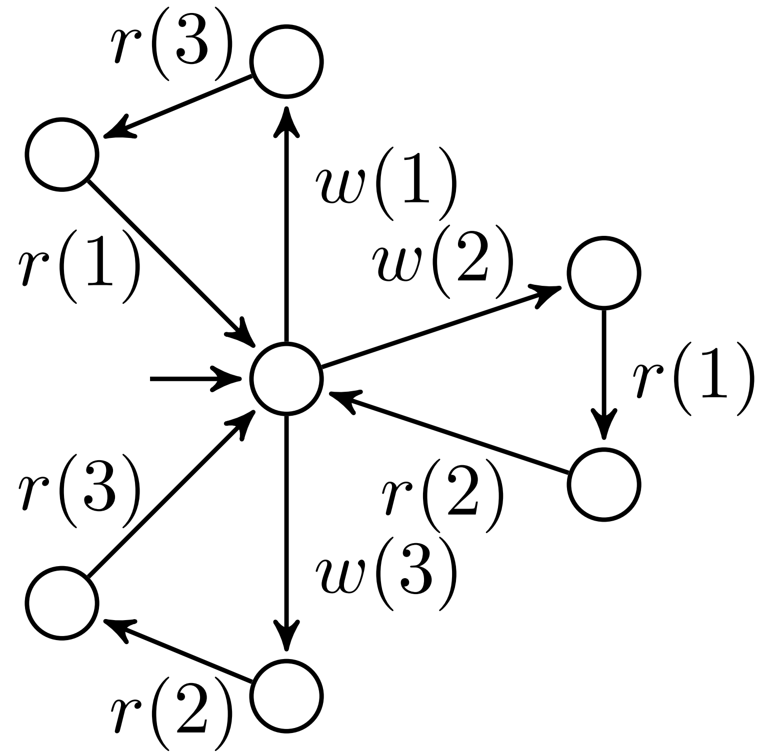


$r(1)$ $r(1)$ $r(1)$ $w(2)$ $r(2)$ $w(1)$ $w(1)$ $r(1)$ $r(1)$ $w(3)$ $r(3)$

Finite abstraction







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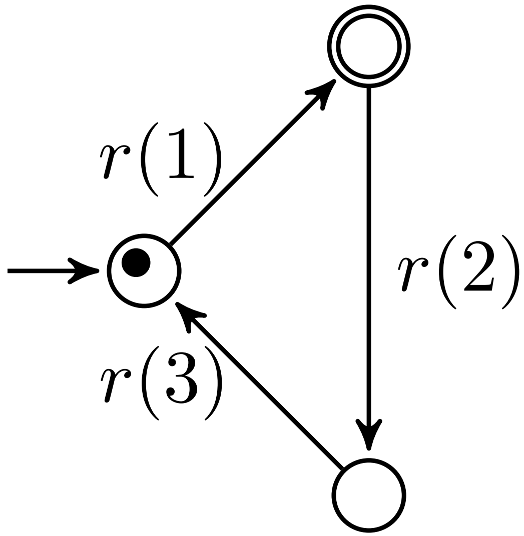

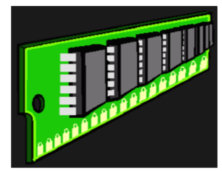

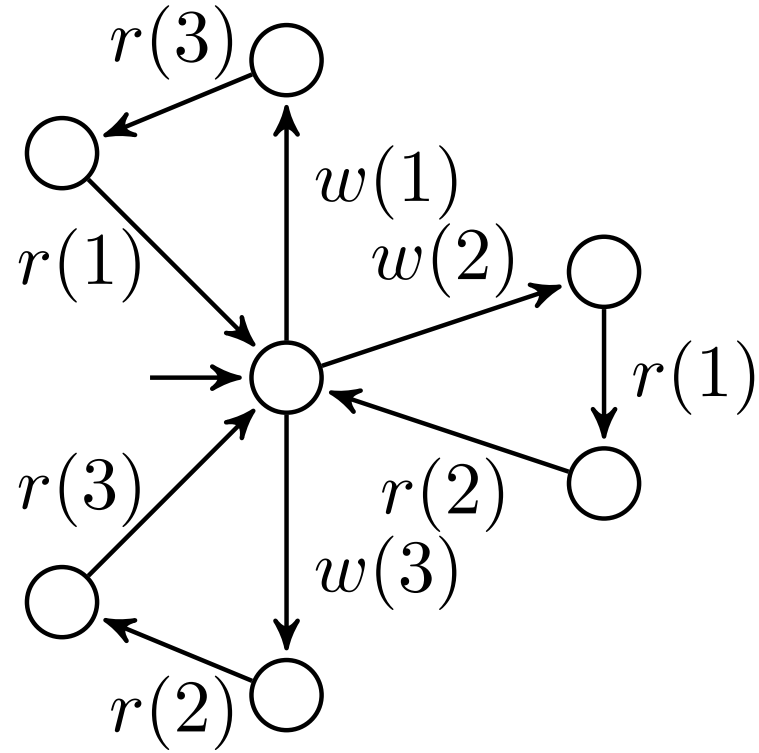

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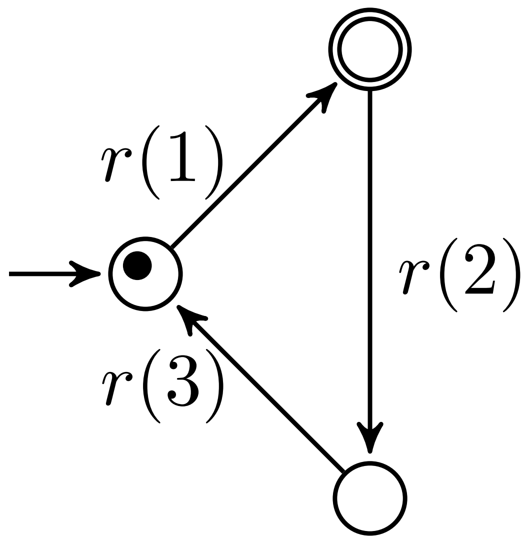

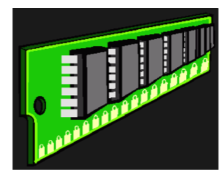

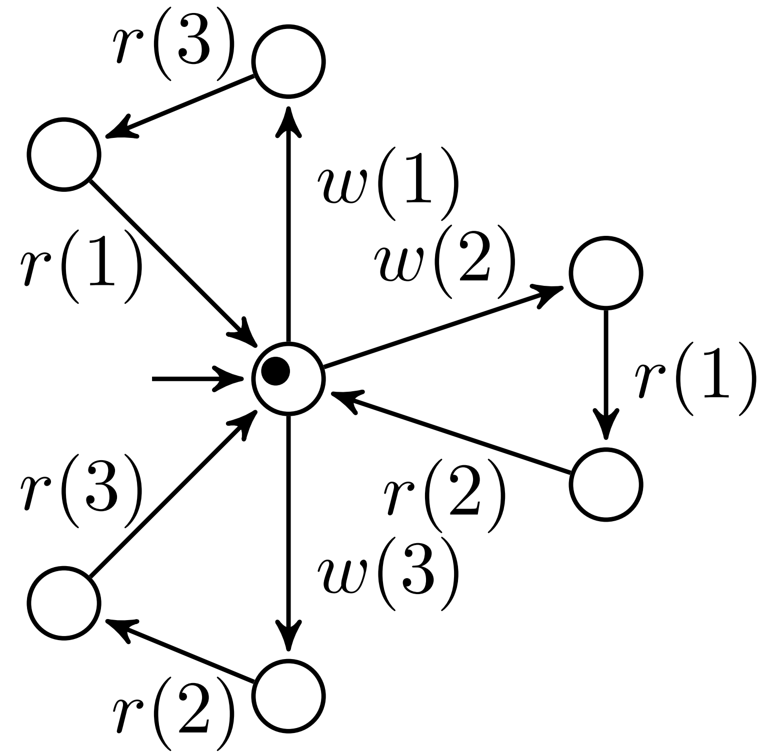

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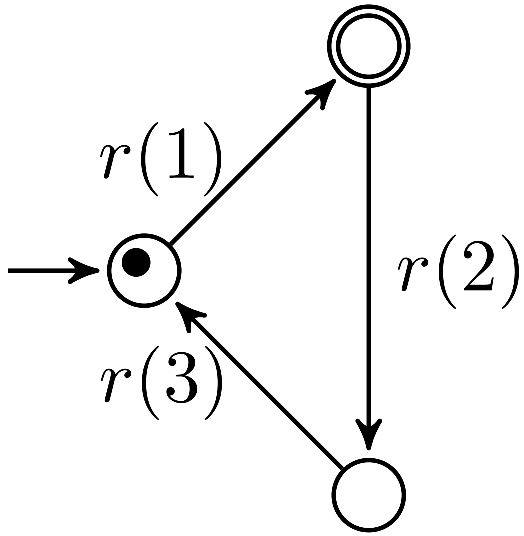

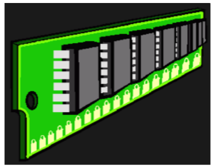

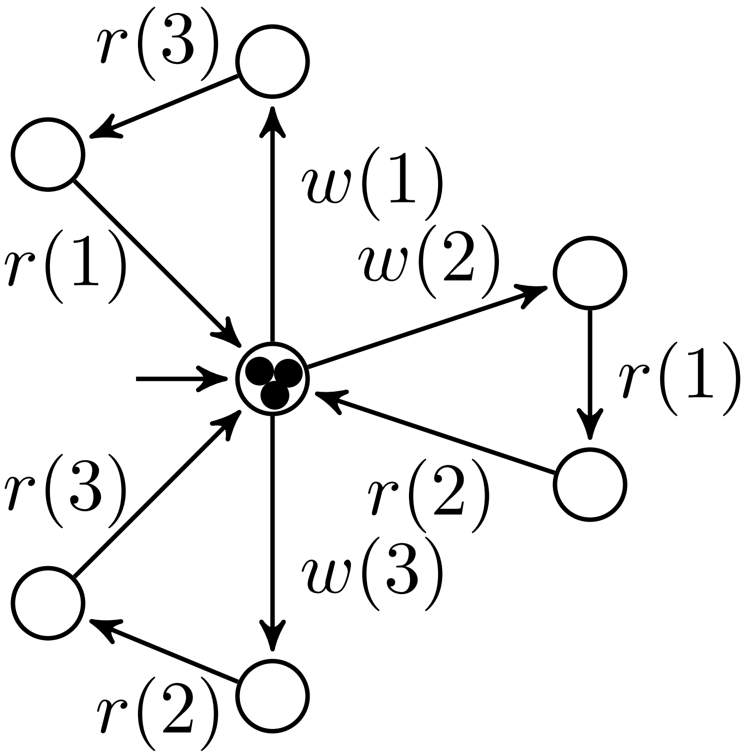

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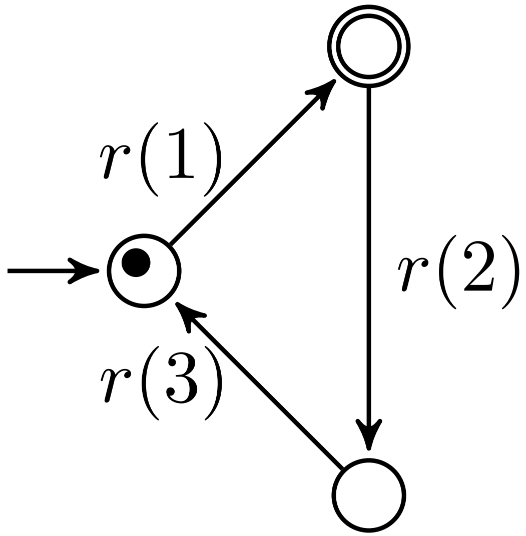

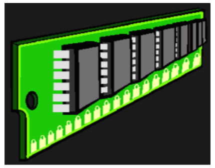

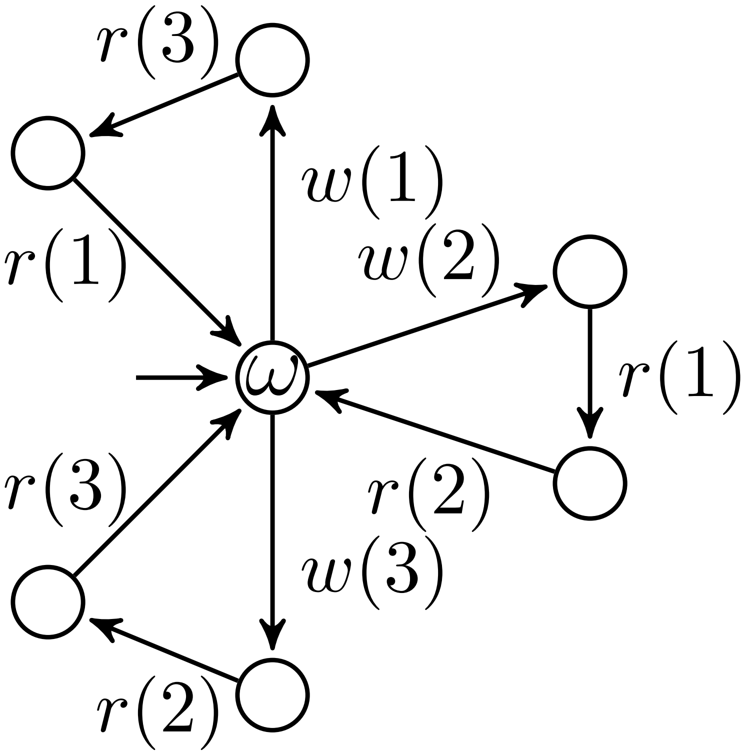

- Headcount of contributors not needed: $\bullet \to \omega$

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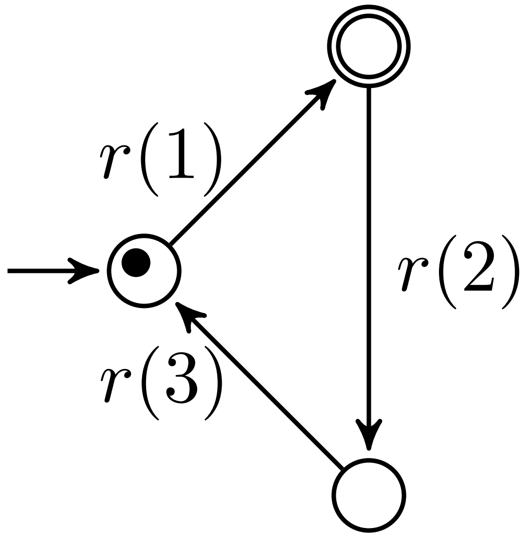

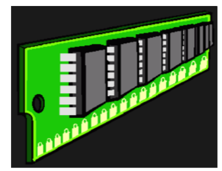

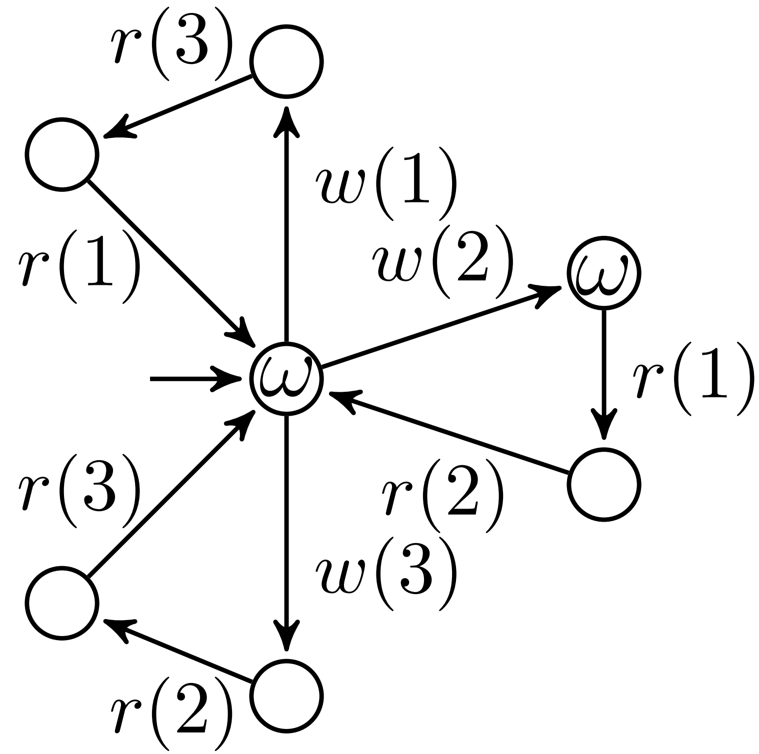

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- Never delete an ω (Copycat lemma)

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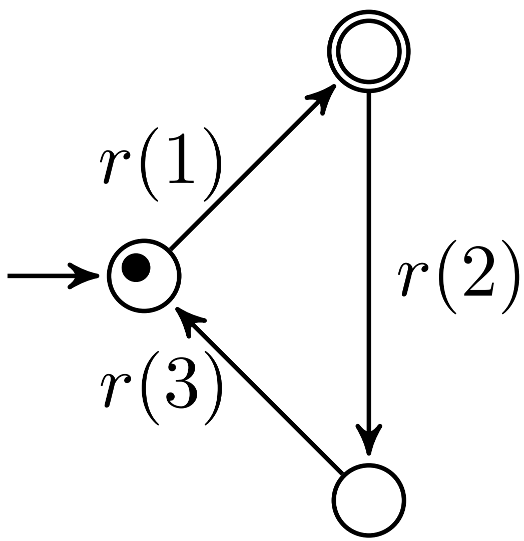

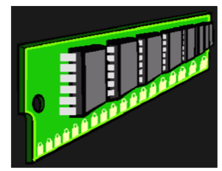

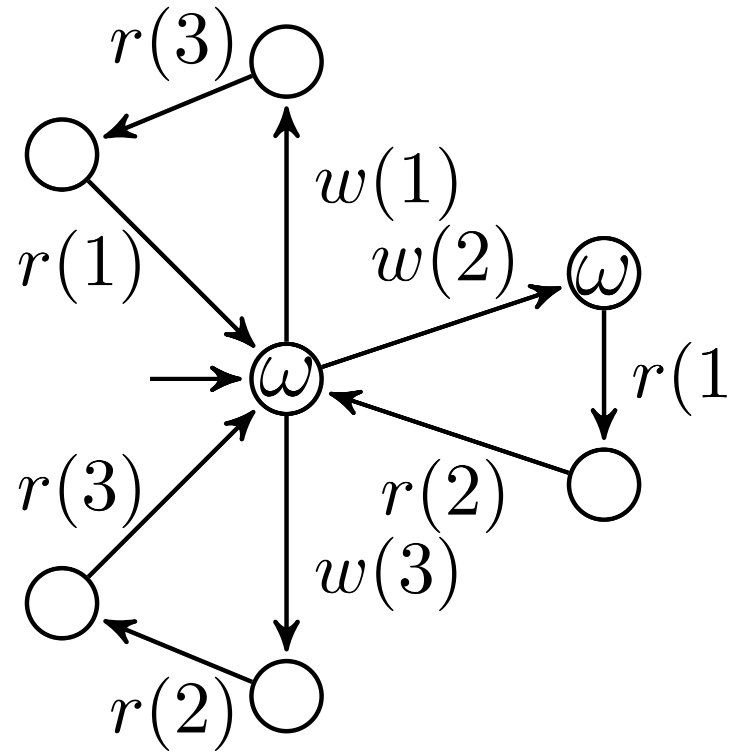

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Build a graph αTS whose nodes reads as:

$\langle$ leader state , store value , { contribs states with ω }$\rangle$

edges are defined following leader or contributors moves

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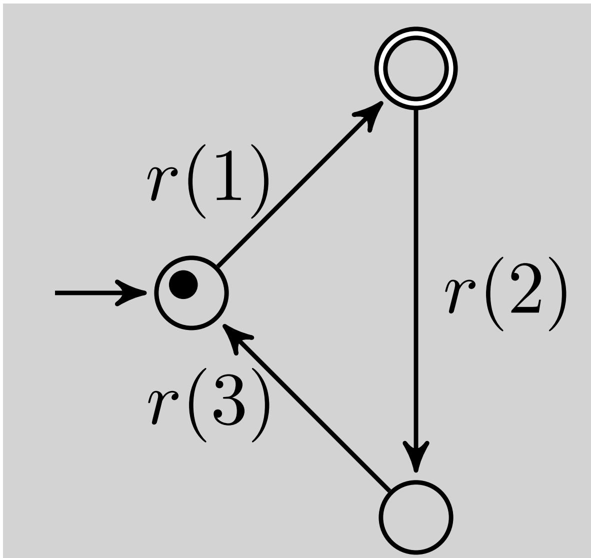

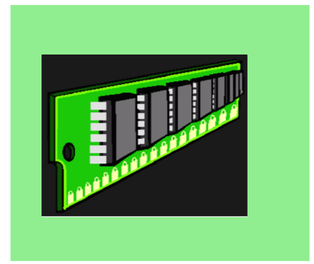

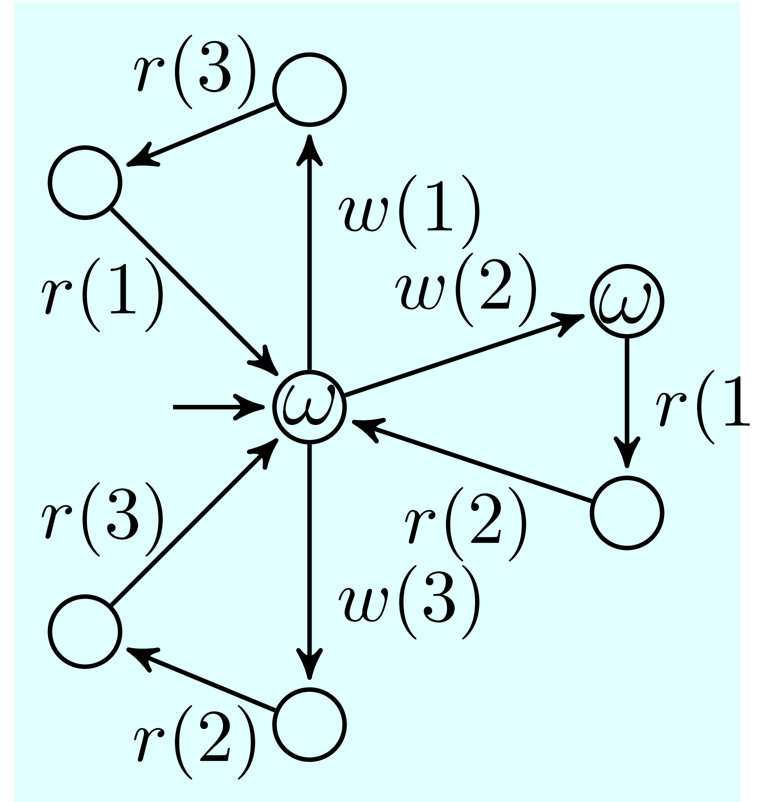

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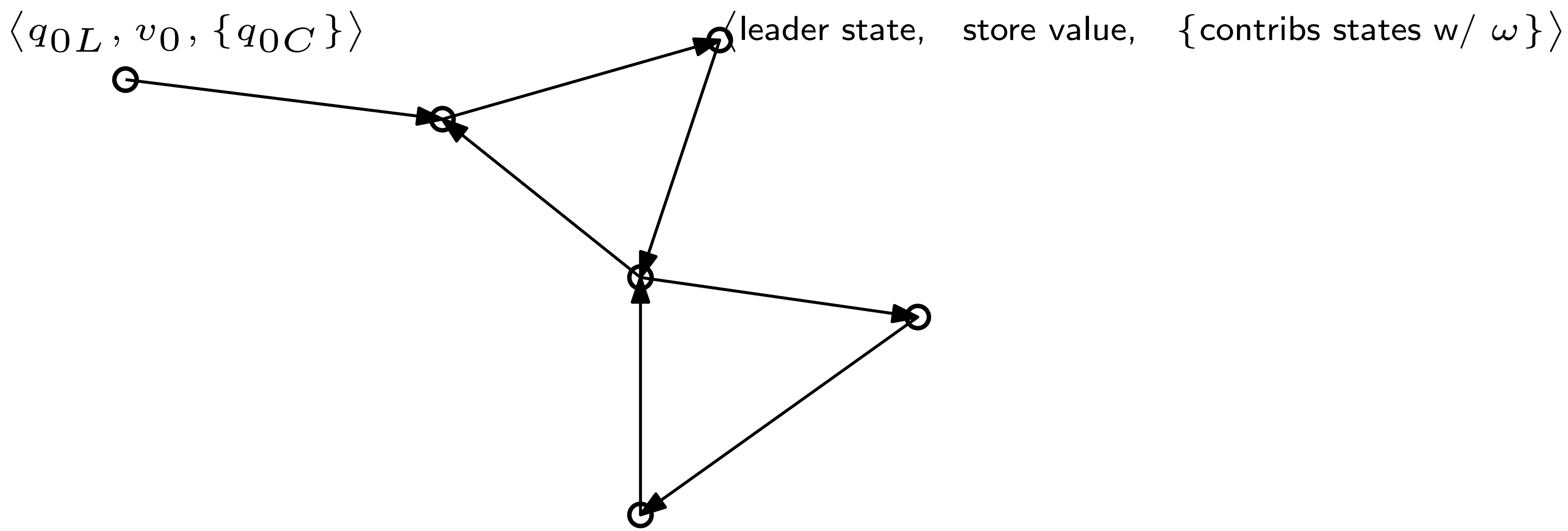
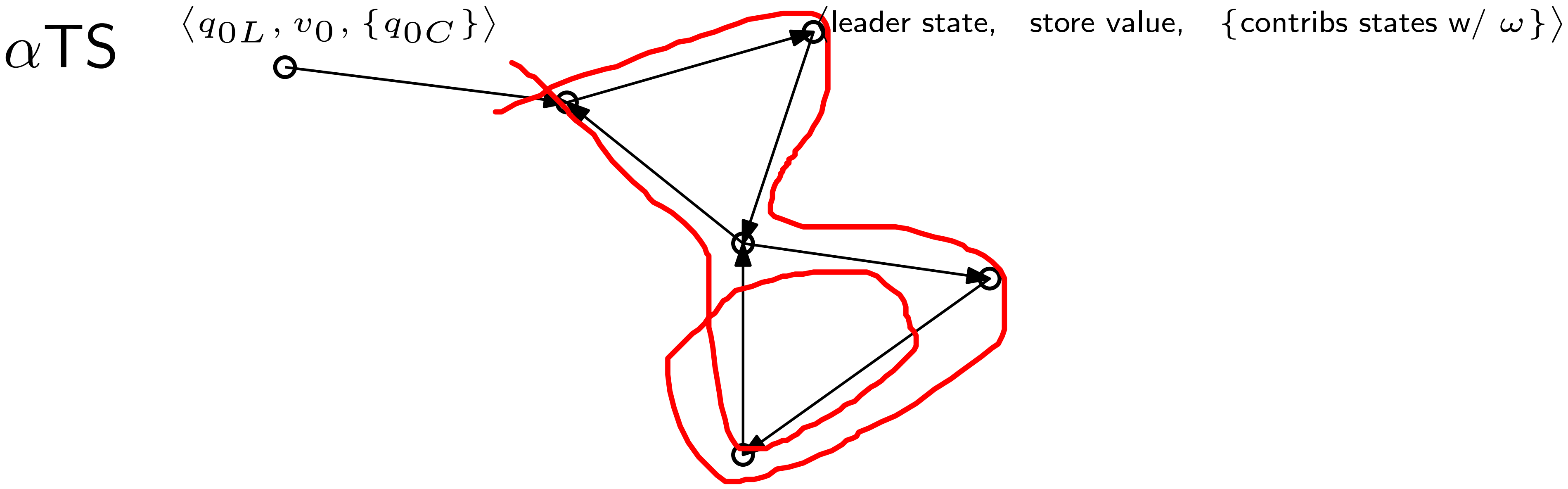
αTS



Decision procedure

Is there an infinite accepting run with finitely many processes?



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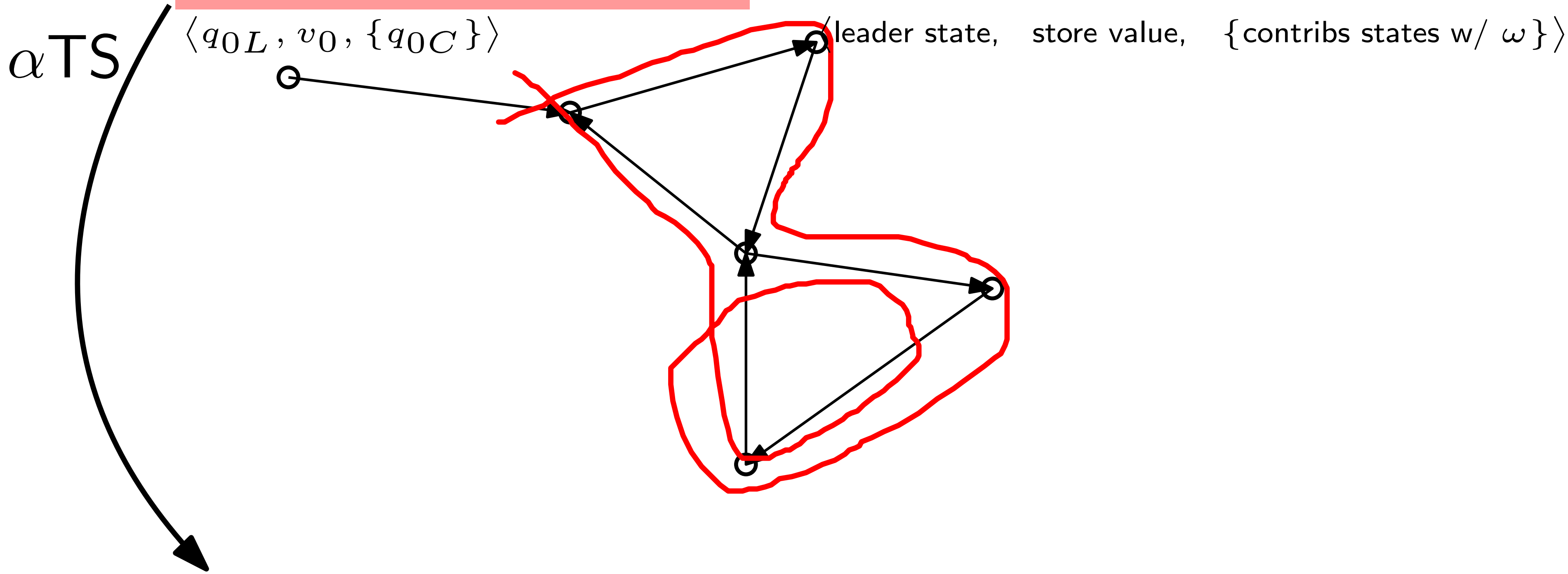

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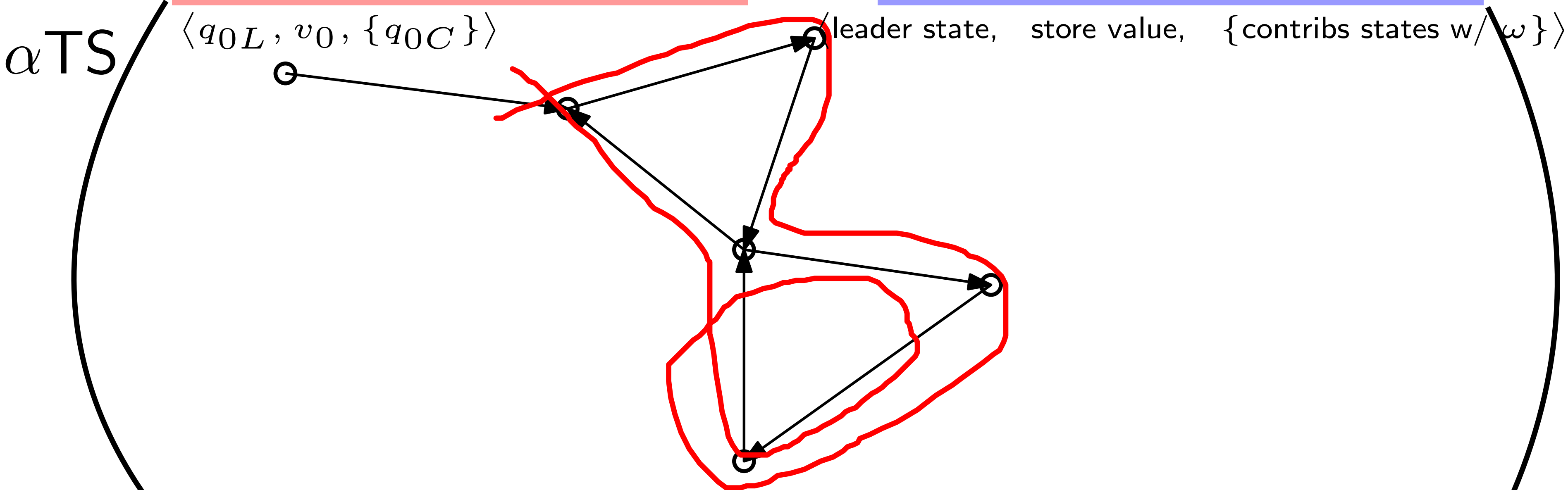


$\exists$ reachable cycle $\cap$ Buchi states $\neq \emptyset$

each contributor involved makes a roundtrip

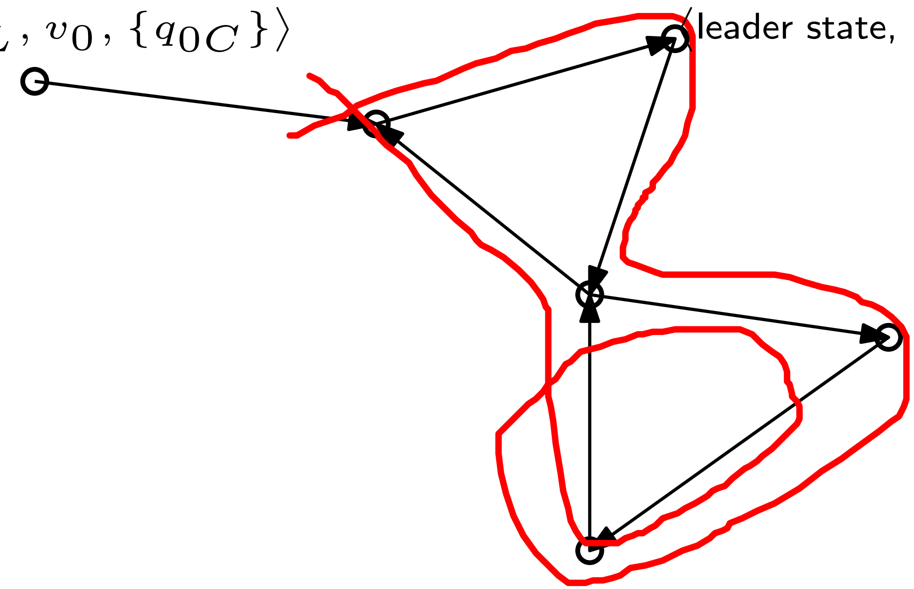
iff

the cycle has $\vec{0}$-weight

Decision procedure

Is there an infinite accepting run with finitely many processes?

αTS



Has αTS a reachable $\vec{0}$-weight cycle visiting Buchi states?

$\Downarrow$

Satisfibility of an existential Presburger formula, in NP

$$\Omega^{\circlearrowleft}(x_{t_1}, \ldots, x_{t_n}) \wedge \bigwedge_{q_c \in Q_c} \left(\sum_{tgt(t)=q_c} x_t = \sum_{src(t)=q_c} x_t \right)$$

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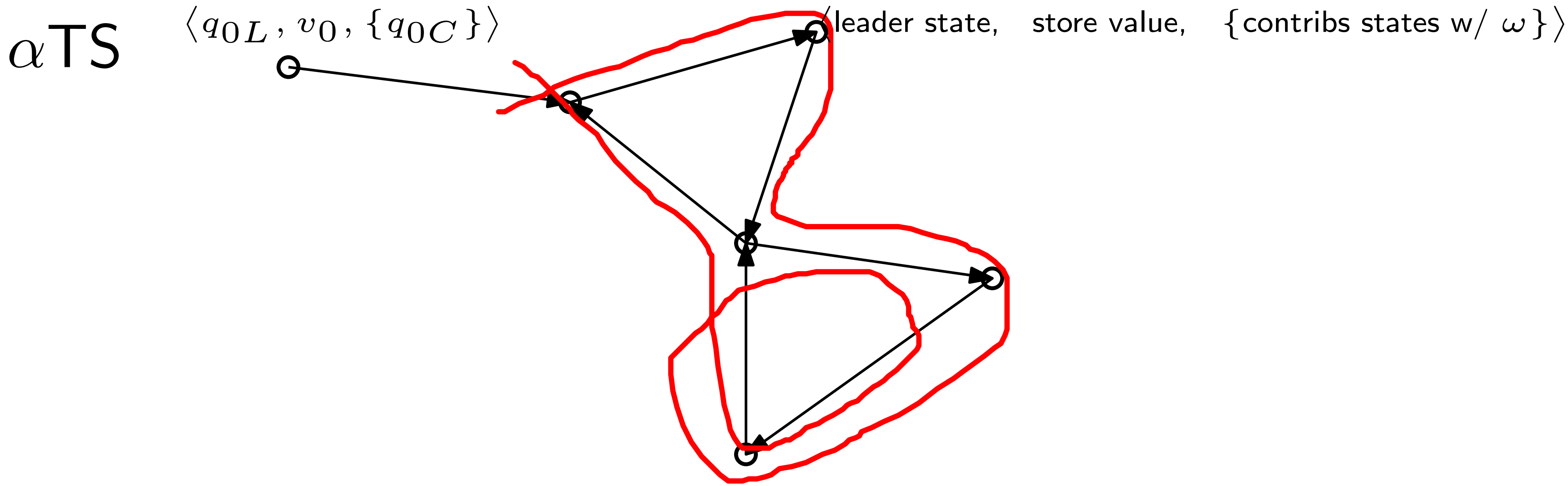

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Parikh Image for finite state automaton

[Seidl et al., ICALP'04]

Wake up!

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I did a mistake

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This is not a polynomial!

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There are exponentially many nodes

$\langle$ leader state, store value, $\{$ contribs states with ω $\}\rangle$

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Obs: along every path of αTS, the states with ω can only grow

Step 1: compute the subgraph for a guessed growing sequence
Step 2: compute the formula and check for SAT

The NP membership in perspective

The **non-parameterized** variant is PSPACE-hard [Kozen,FOCS'77]

Arbitrarily many processes yields a **noisy** channel and the problem gets easier

The results show that **noise** can **not** be cancelled out

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What if we give processes some auxiliary memory?

Processes + stacks

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- To the leader only:

  remains in NP (proof generalization)

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Compare with the undecidability result for **fixed** number of processes **all** of them with stack

Only the leader has a stack

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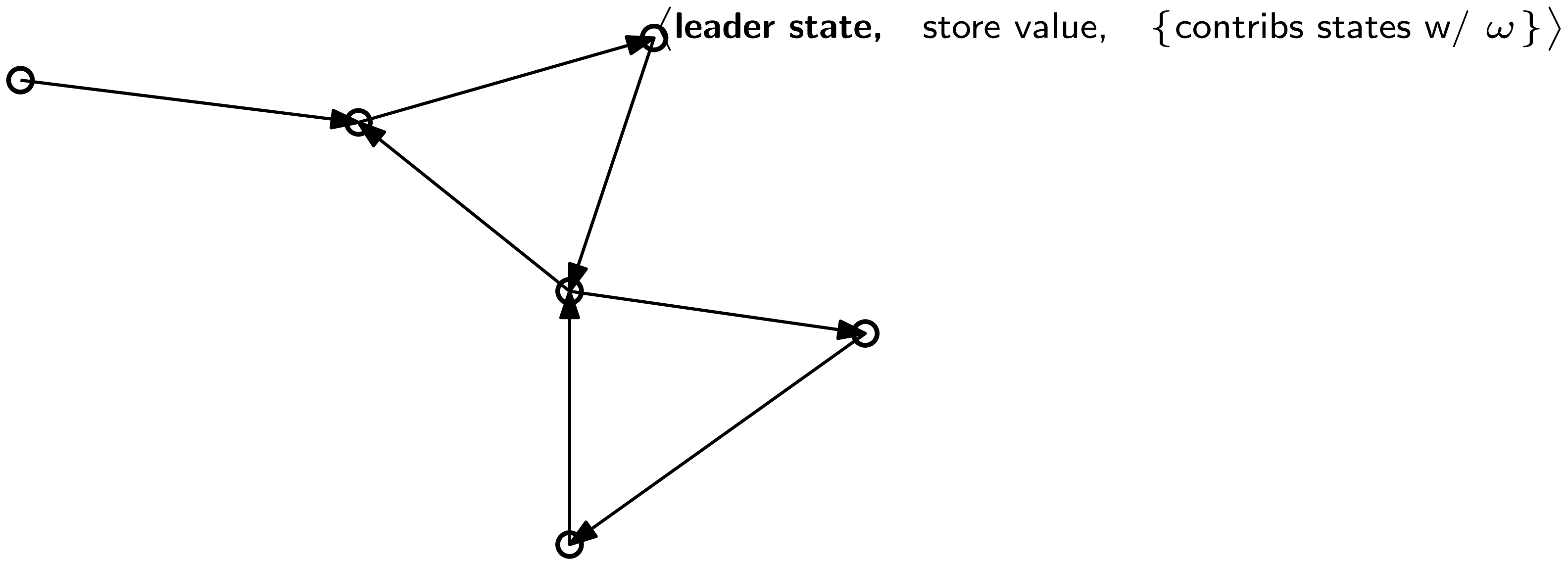

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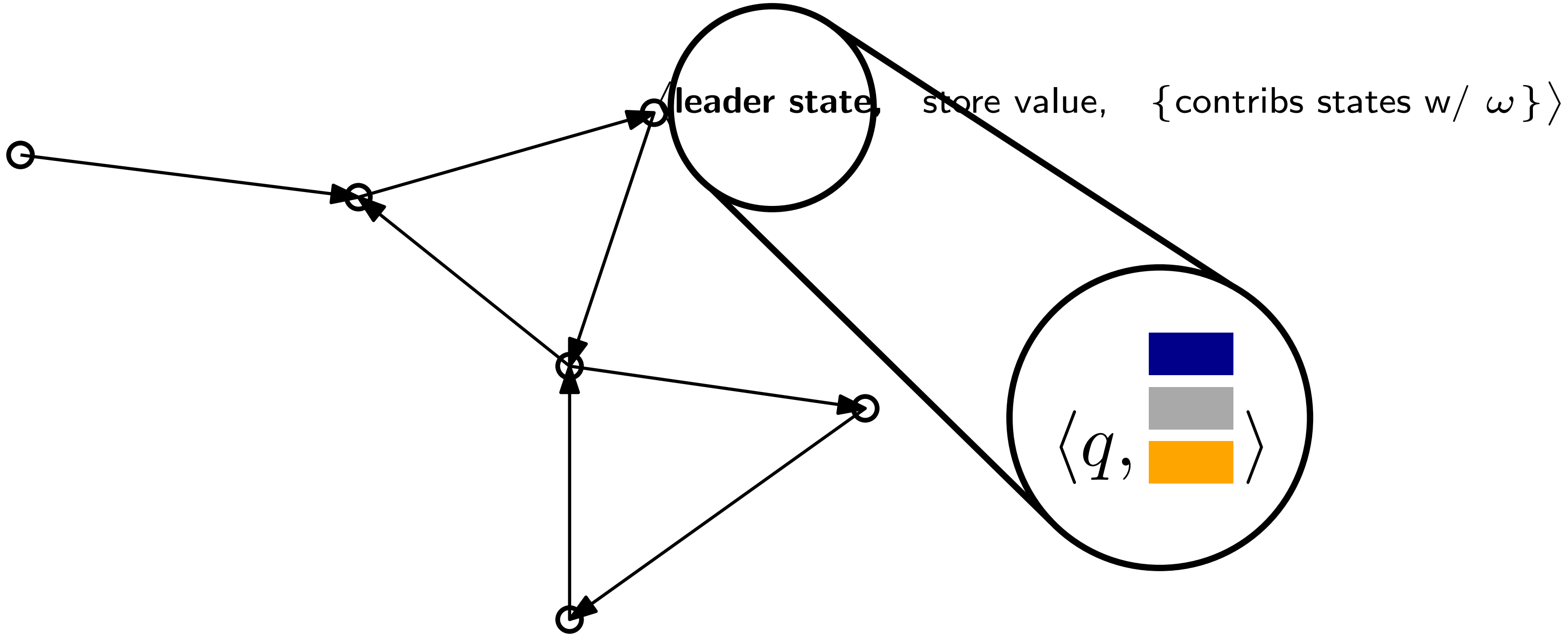

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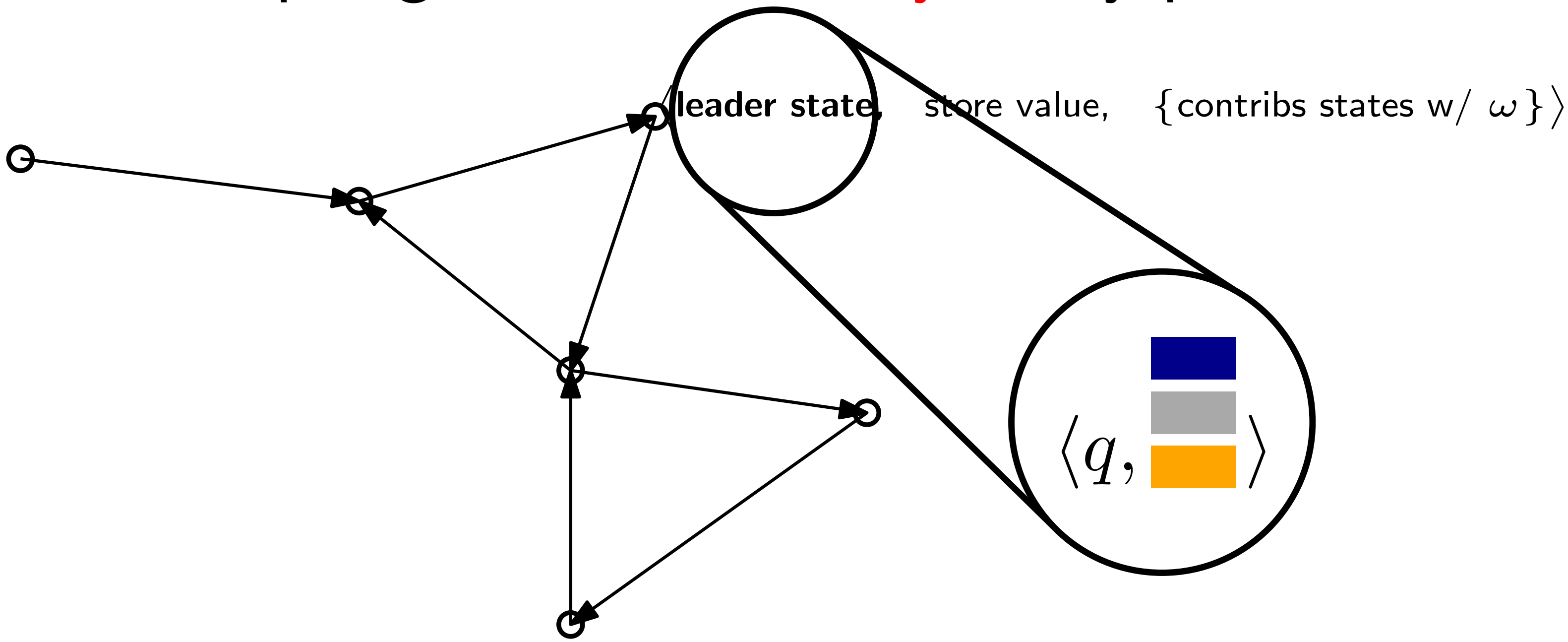

Has αTS a reachable $\vec{0}$-weight cycle visiting Buchi states ?

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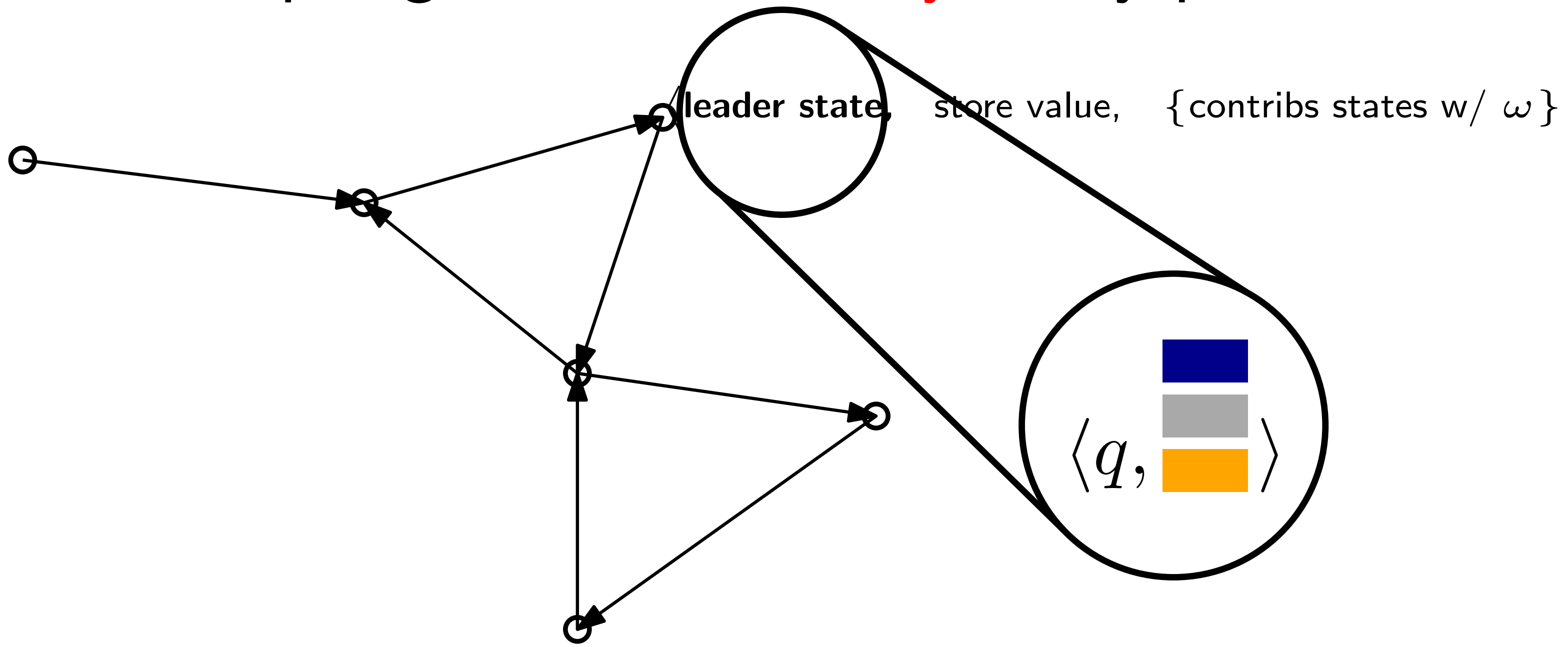

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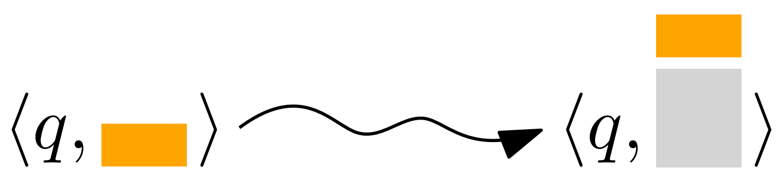

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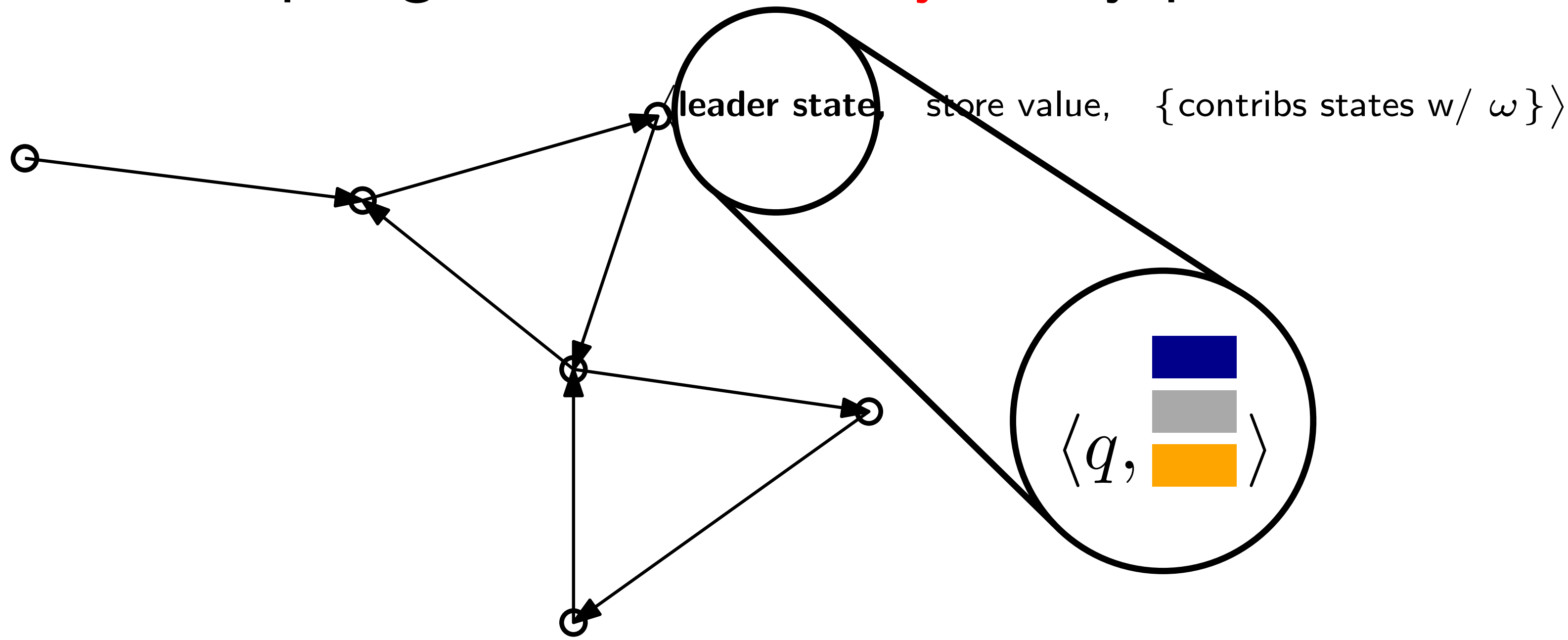

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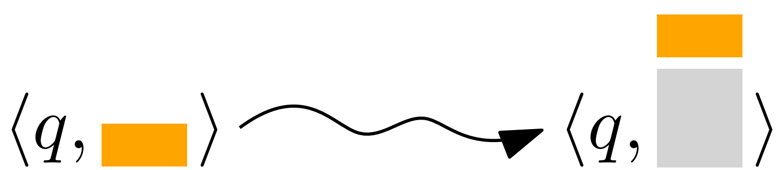

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Parikh Image for context-free grammars

[Seidl et al., CADE'05][Esparza, Fun. Informatica'97]

Everybody has a stack

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NEXPTIME

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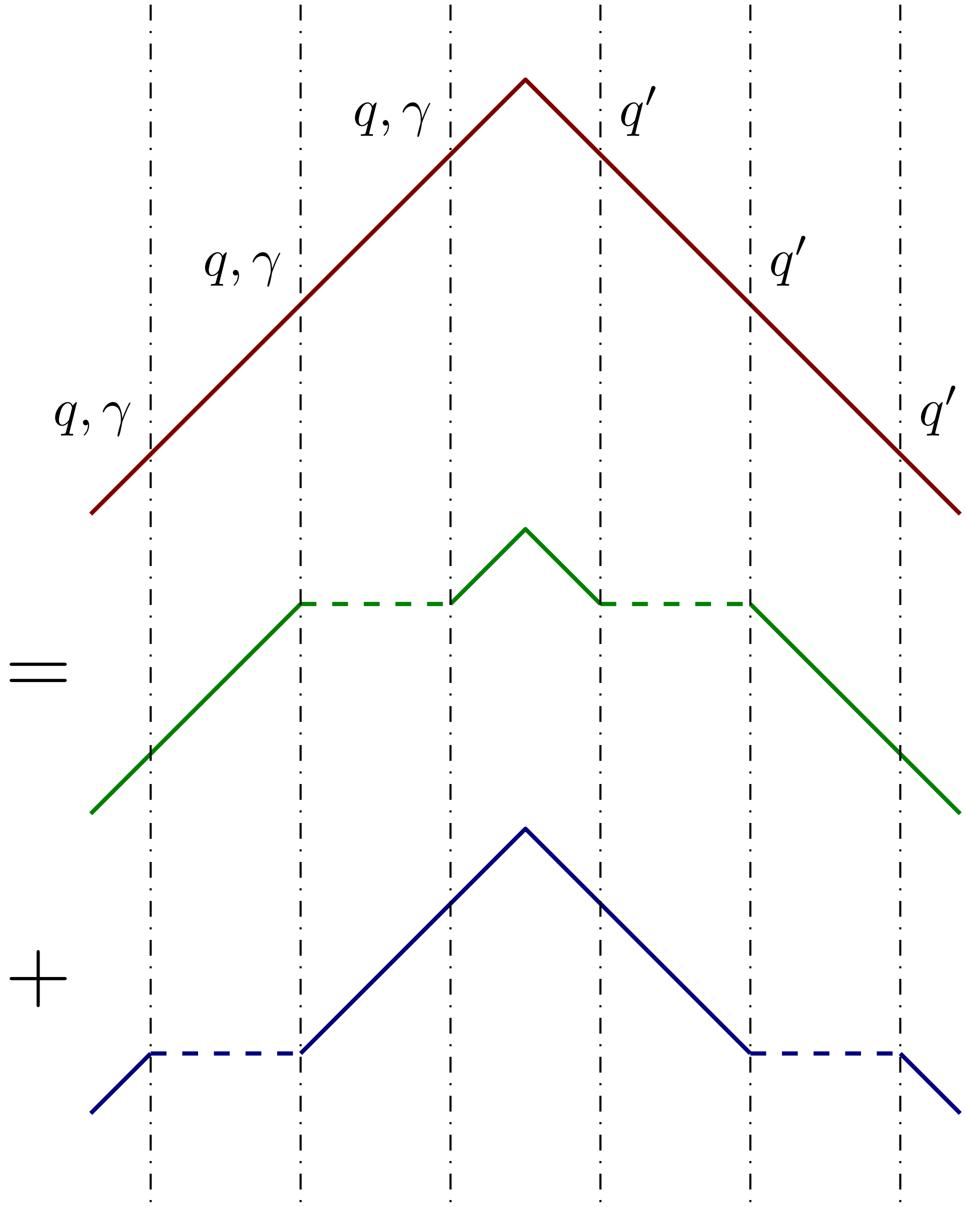
Proof of correctness is not simple or easy

Replace a contributor run by **more** runs using "less memory"



Less memory = retrieve shallow stack frames only

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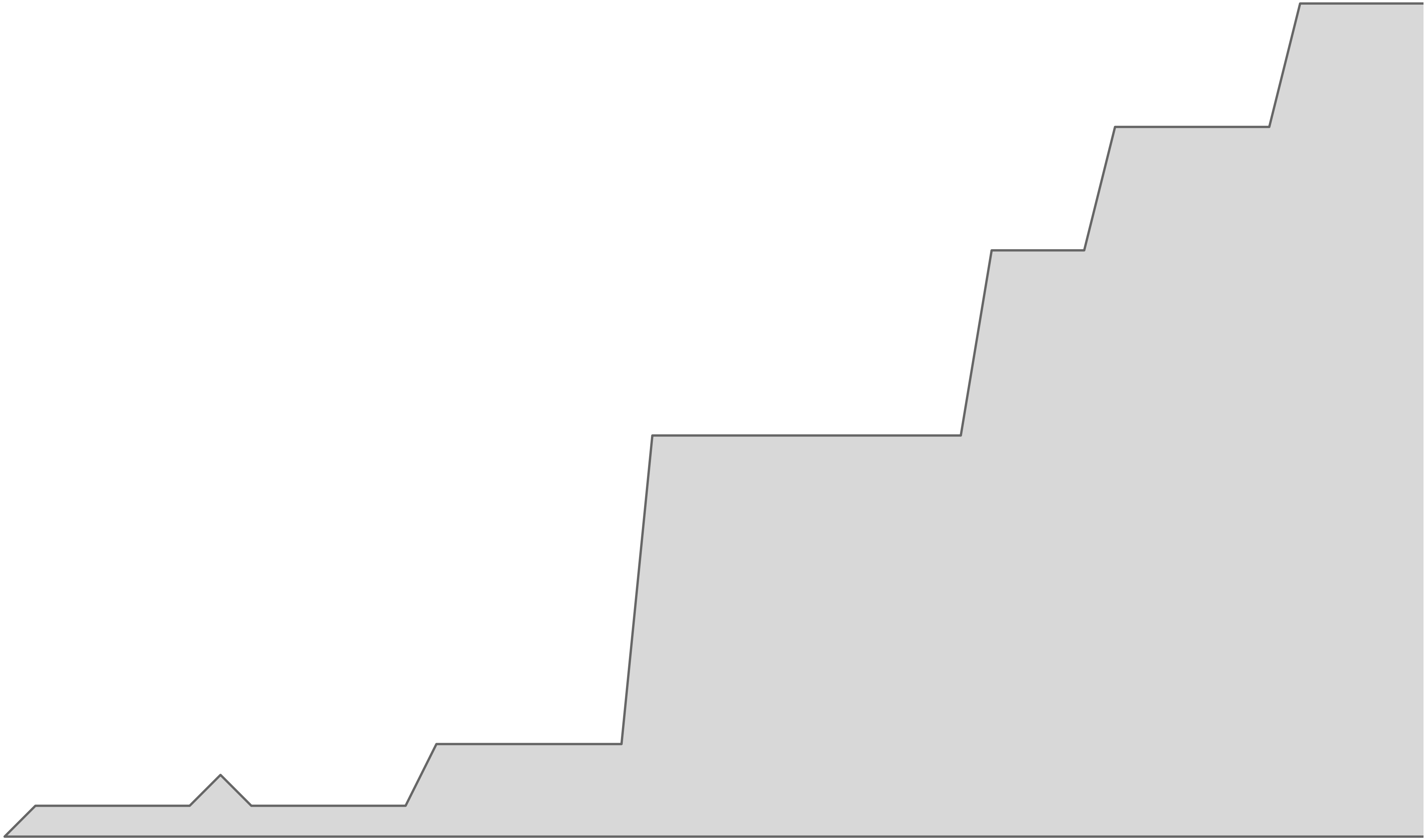
- Examples



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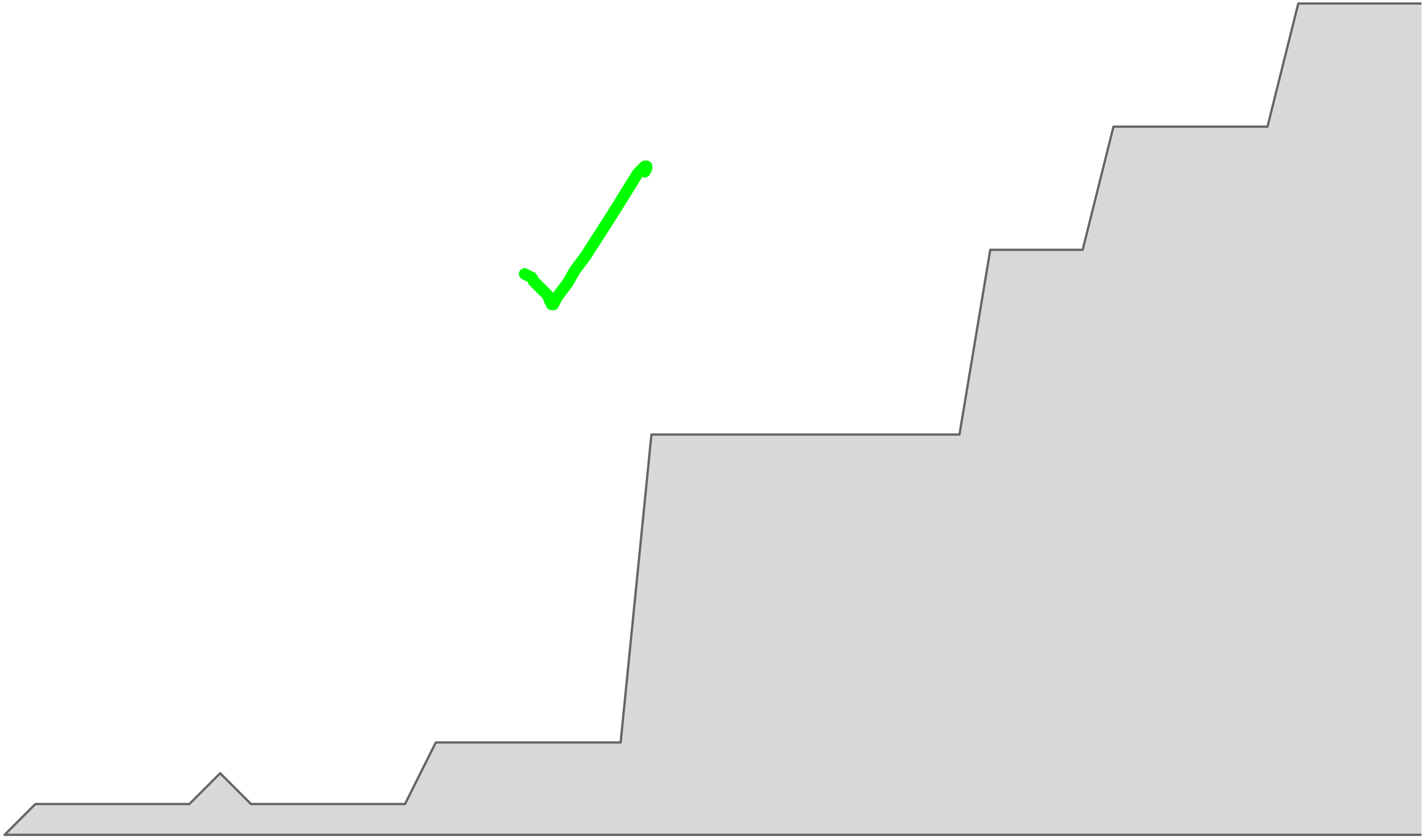

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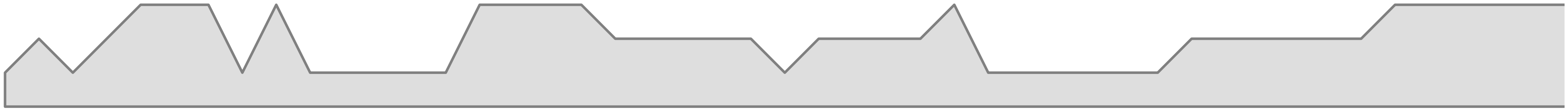

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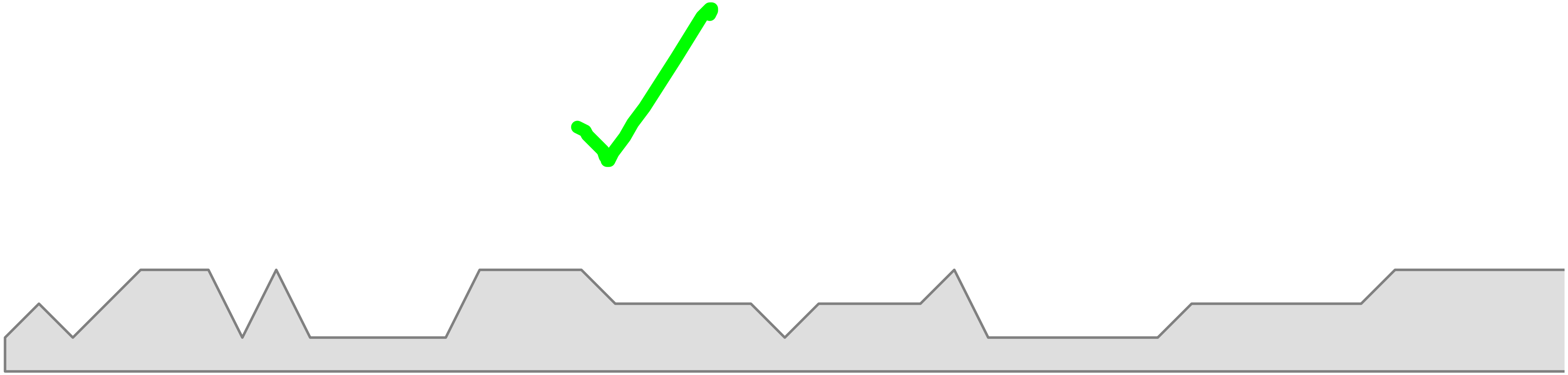

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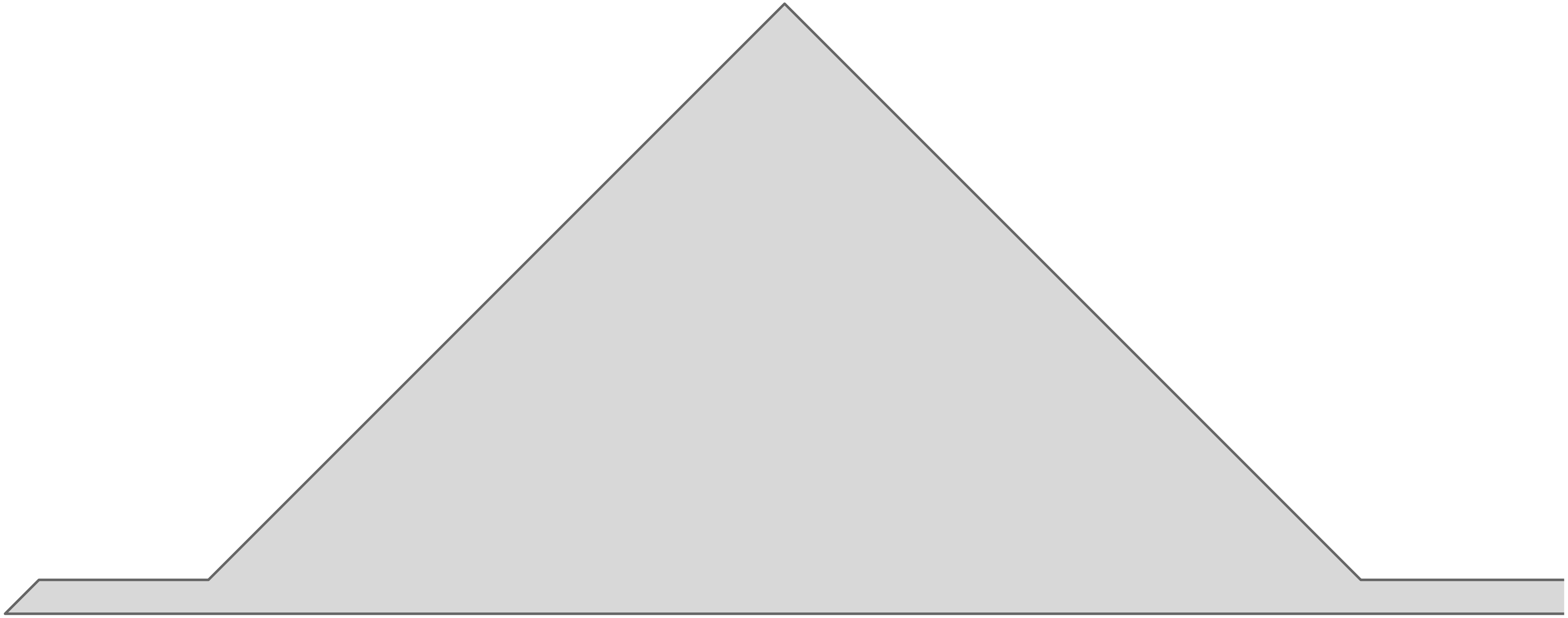

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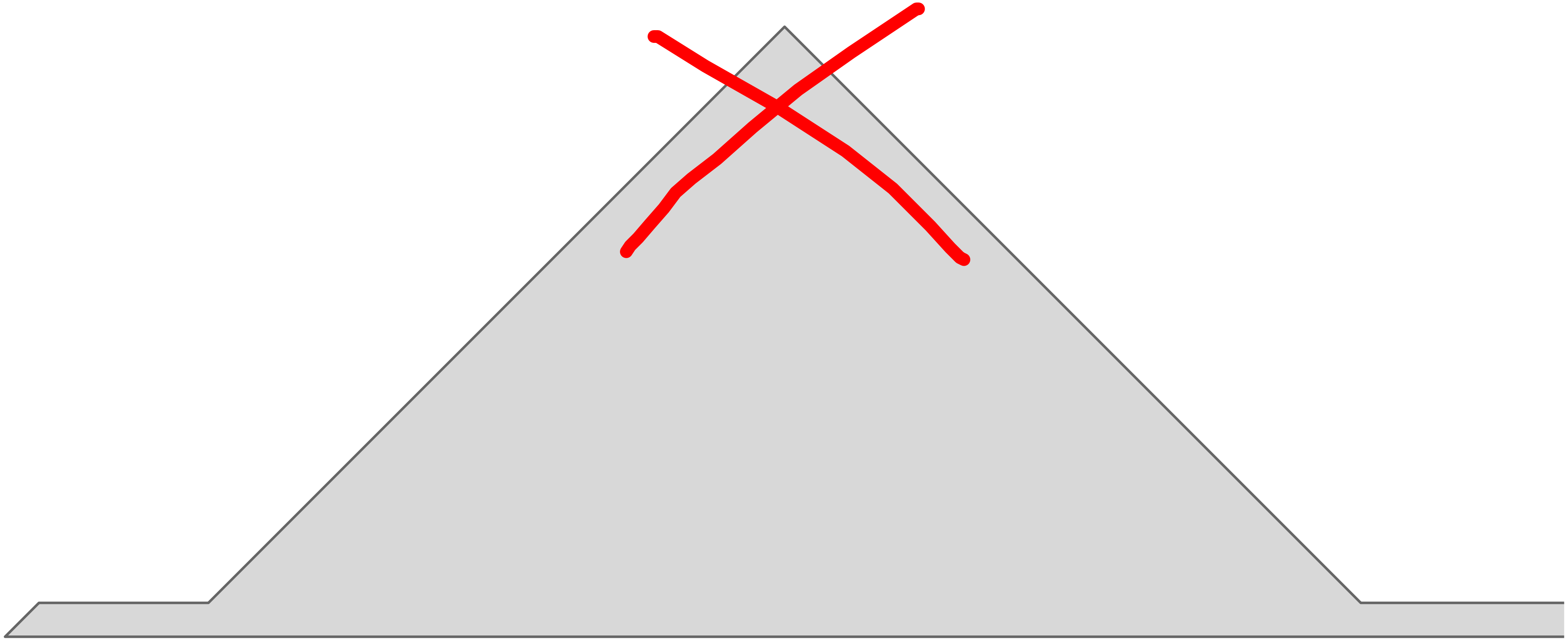

Less memory = retrieve shallow stack frames only

- having each contrib keeping track of its topmost $O(n^3)$ frames suffice

- Memory is now bounded and hard-encoded into the states

- Fall back onto the “only the leader has a stack” case

Conclusions

Safety checking (CAV'13)

	FSM	PDM
FSM	coNP-c	coNP-c
PDM	coNP-c	PSPACE-c

Liveness checking (CAV'15)

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PDM		PSPACE-hard ? NEXPTIME

Liveness is as easy as safety

- Contrast: broadcast protocols

Parameterized verification is easier than non-parameterized

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